MATH5 CLASSWORK 20

March 23, 2025

Recall: Square root of a (denoted $\sqrt{a}$ is a number whose square is equal to a. For example: square root of 25 is 5, because $5^2 = 25$.

We discussed that

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

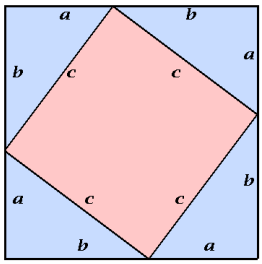
$$\sqrt{a + b} \neq \sqrt{a} + \sqrt{b}$$

Square roots naturally appear in geometry:

Pythagorean Theorem: In a right triangle with legs a, b and hypotenuse c, one has

$$a^2 + b^2 = c^2 \quad \text{or} \quad c = \sqrt{a^2 + b^2}$$

A proof of this theorem is illustrated below:

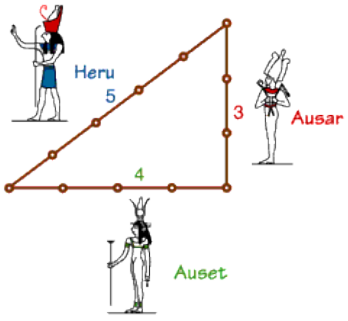


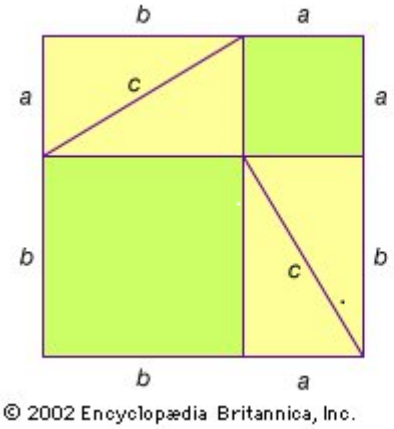
In this square, the total area is

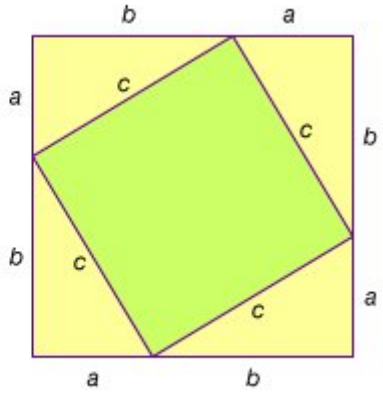
$(a + b) \times (a + b) = a \times (a + b) + b \times (a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

On the other hand, the area of each triangle is $\frac{1}{2}ab$; and the area of shaded square is c^2. Thus, we get $a^2 + 2ab + b^2 = 4 \cdot \frac{1}{2}ab + c^2$, which gives

$$a^2 + b^2 = c^2$$







MATH5 HOMEWORK 20

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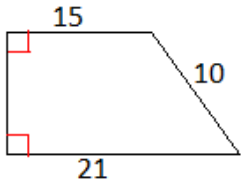
Please make sure your solutions for the problems with ◿ are accompanied by a picture done with a ruler and a pencil. Drawing a problem will help you to solve it.

1. ◿ Can you find a right triangle where all sides are whole numbers and the hypotenuse is 13?
2. ◿ If, in a right triangle, one leg has length 1 and the hypotenuse has length 2, what is the other leg?
3. Find $\sqrt{2^6 \cdot 7^2}$ [use square root properties we discussed]; $\sqrt{\frac{1}{16}}$; $\sqrt{\frac{4}{9}}$.
4. ◿ Find the height and area of the figure below. Lengths of three sides are given; the two marked angles are right angles.



5. **Take some positive number x < 100** and using calculator (or computer) calculate the number $\frac{x}{2} + \frac{1}{x}$. Call the result x and repeat the same calculation with the new x. Do it 10 times. We will compare the results in class.

x	$\frac{x}{2} + \frac{1}{x}$
Initial x	result_1
result_1	result_2
result_2	result_3
result_3	…….
…….	…….
result_9	result_10

6. What did the artist want to tell? Use mathematical language to explain, name what you need.