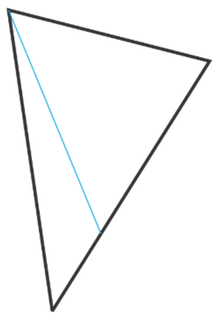
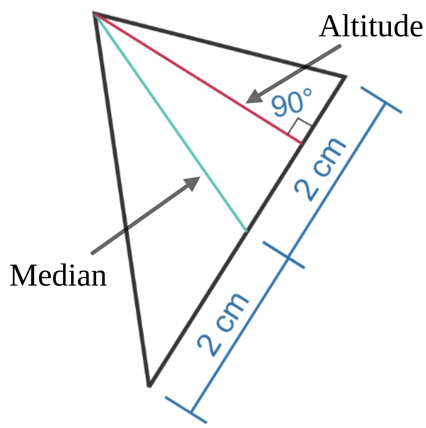
Math 5b. Classwork18

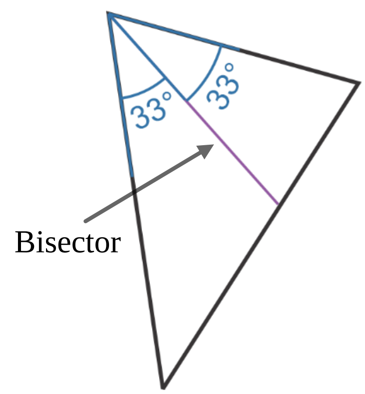


In each triangle we can connect a vertex with a point on the opposite side, like on the picture. Infinitely many segments can be drawn from any vertex to an opposite side, but there are three very special segments. From each vertex one can draw a segment to a midpoint of the opposite side, such segment is called a ***median***.

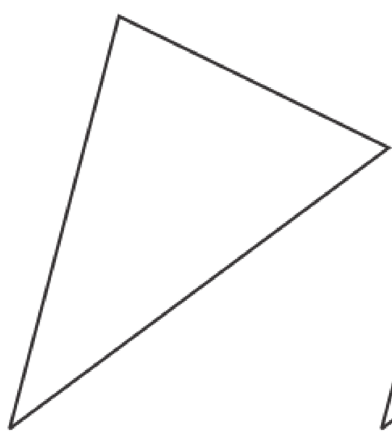
Also, from any vertex one can draw a perpendicular to an opposite side (or to continuation of the opposite side in the case of obtuse angle). This segment is called an ***altitude*** (or height).

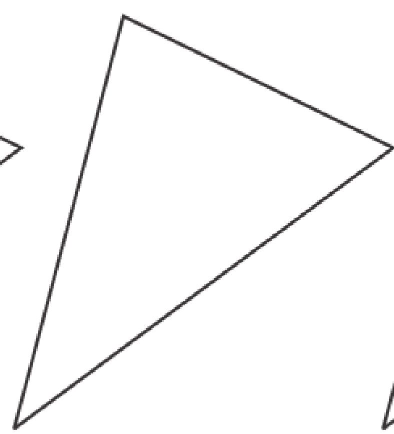
And the last segment is an (angular) ***bisector***. It’s a segment, drawn from any vertex of a triangle, in a way that the angel is divided into two equal angles.

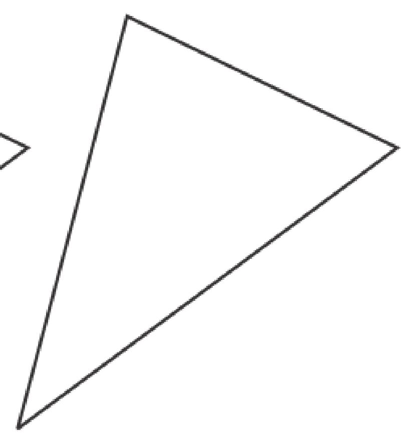


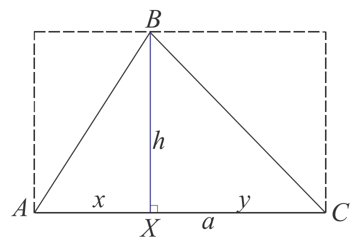


Draw medians, altitudes and bisectors in the triangles:







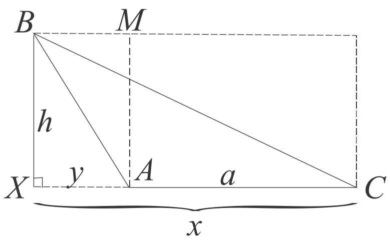


The area of a triangle is equal to half of the product of its altitude and the base, corresponding to this altitude.

For the acute triangle it is easy to see.

$S_{rec} = h \cdot a = h \cdot (x + y) = hx + hy$

$S_{\Delta ABX} = \frac{1}{2}h \cdot x, \qquad S_{\Delta XBC} = \frac{1}{2}h \cdot y, \qquad S_{\Delta ABC} = S_{\Delta ABX} + S_{\Delta XBC}$



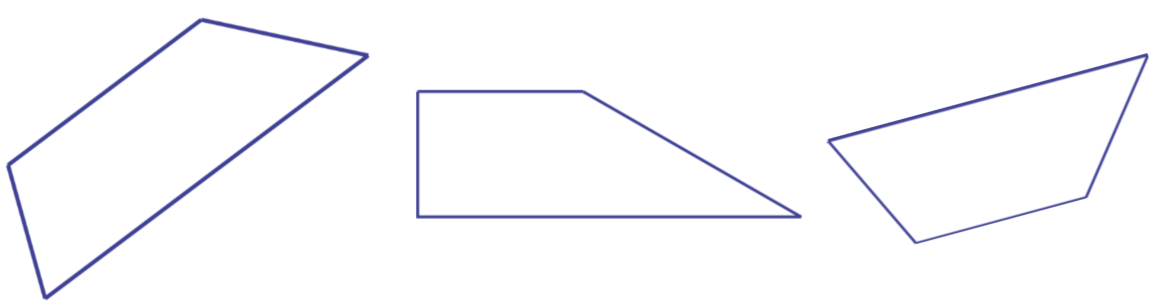
$$S_{\Delta ABC} = \frac{1}{2}h \cdot x + \frac{1}{2}h \cdot y = \frac{1}{2}h(x + y) = \frac{1}{2}h \times a$$

For an obtuse triangle it is not so obvious for the altitude drawn from the acute angle vertex. Can you come up with the idea how we can prove it for an obtuse triangle? Area of trapezoid?

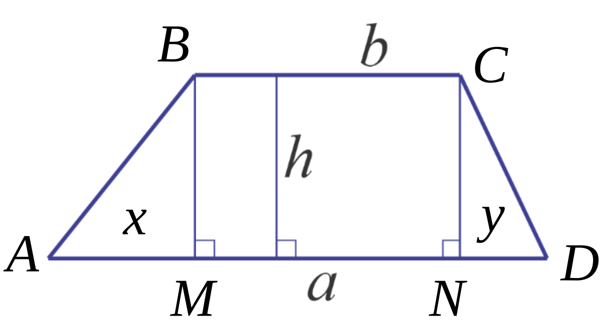
Quadrilaterals.

Polygons with four sides and four vertices are quadrilaterals. Quadrilaterals can have four non parallel sides, two parallel and two not parallel sides, and two pairs of parallel sides.

If the quadrilateral has only one pair of parallel sides and two other sides are not parallel are called trapezoids.



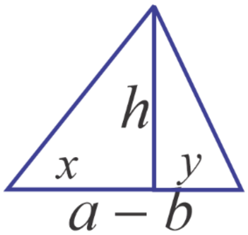
Trapezoid has two bases, a and b, they are parallel segments. h is an altitude (height), segment, perpendicular to bases. How to find area of the trapezoid? MBCN is a rectangle, area of this rectangle is

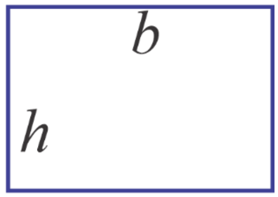


$$S_{rectangle} = h \cdot |MN| = h \cdot b$$

Area of the trapezoid is

$$S = S_{rectangle} + S_{AMB} + S_{NCD}$$





$$S_{AMB} + S_{NCD} = \frac{1}{2} \cdot h \cdot (x + y) = \frac{1}{2} \cdot h \cdot (a - b)$$

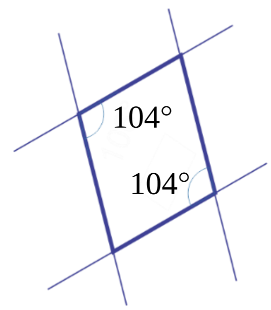
$$S = S_{rectangle} + S_{AMB} + S_{NCD}$$

$$= hb + \tfrac{1}{2}h(a - b) = hb + \tfrac{1}{2}ha - \tfrac{1}{2}hb = \tfrac{1}{2}hb + \tfrac{1}{2}ha = \tfrac{1}{2}h(a = b);$$

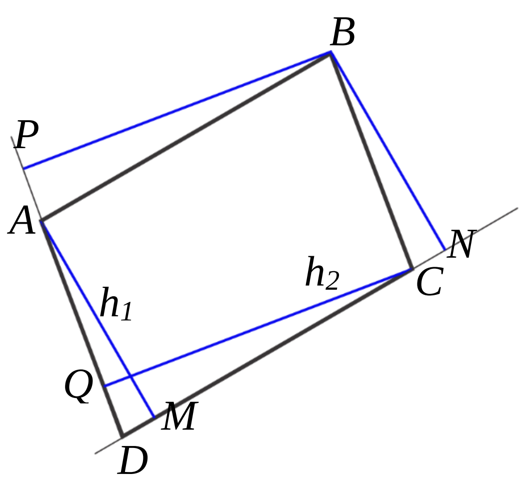
Area of a trapezoid is a half of the product of the altitude and the sum of the bases.

If a rectangle has two pairs of parallel lines it's called a parallelogramm. Parallelogramms havs e few properties:

- Their sides not only parallel, but also equal.
- Diagonal divides a parallelogram onto two equal (congruent) triangles.
- Diagonals intersect at the midpoint.
- Opposite angles are equal.



How do we call a parallelogram with all right angles? Parallelograms with equal sides are called rhombuses.



Area of a parallelogram. On the picture below. ABCD is a parallelogram. Segments [AM] and [BN] are equal and perpendicular to lines (DC) and (AB). Triangles DAM and CBN are equal. You can see it by superimposing them (and it can be proved based on the theorems of triangle equalities). So the area of parallelogram is equale to the area of a rectangle

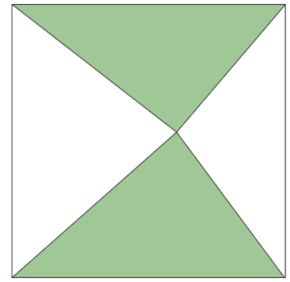
$$S_{ABNM} = |MN| \cdot h_1 = |DC| \cdot h_1$$

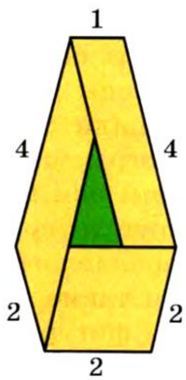
(h_1 is an altitude, distance between a pair of parallel lines. Of course, it's also equal to

$$S_{ABNM} = |AD| \cdot h_2$$

Exercises:

1. Which part of the square is shaded?

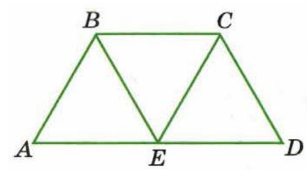




2. Draw two different parallelograms with diagonal, equals to 4 cm and 6 cm.

3. All quadrilaterals on the picture are parallelograms. What are the lengths of the sides of green triangle?

4. Quadrilateral is not a parallelogram, but it has one pair of parallel sides and one pair of equal sides. Draw this quadrilateral.



5. ABCE and BEDC are rhombuses. |BC| is 3 cm. Find the perimeter of triangle BEC. What are the angles of this triangle.

6. Diagonals of a rectangle are equal, diagonals of a square not only equal but also perpendicular. Draw two different rectangles with diagonal 6 cm. Draw a square with diagonal 6 cm. Is it possible to draw another square with a diagonal length of 6 cm, which is not equal to the first one?

7. Points A, B, and C are vertices of a parallelogram. Draw all possible parallelograms.

8. Draw a rhombus with diagonals 4 cm and 6 cm.

9. Find the sum of

$$\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\cdots+\frac{1}{2024\cdot 2025}$$

10. Write without parenthesis;

$$a.\ \ (x-1)(x+1); \qquad\qquad b.\ \ (x-1)^2;$$

11. Using the result from the previous problem, reduce the fraction:

$$\frac{x^2-2x+1}{x^2-1}; \qquad x \neq 1, -1$$

12. Points A, B, and C are vertices of a parallelogram. Draw all possible parallelograms.