Math 5b. Classwork 8.

Ratio of the green balloon to the red balloons is $\frac{1}{4}$ (1 to 4). On both pictures there are four red balloons for each green balloon. The number of green balloons is a quarter of the number of red balloons. (Or, the number of RB is 4 times greater than number of GB).

$$\frac{1}{4} = \frac{4}{16}$$

Problem1:There are 1165 red and green balloon in the store. The ratio of GB to RD is 1 to 4. How many green and red balloon are in the store? We can solve this problem by two methods:

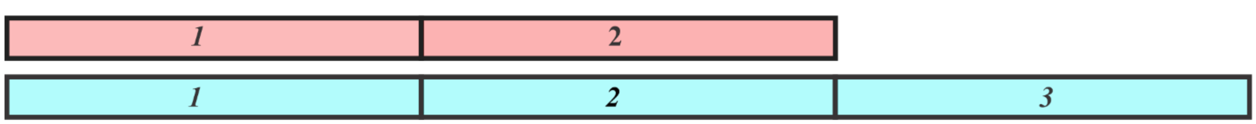
1. Because there are 4 RB for each green one, all balloons can be divided into groups of 5 – 1 green and 4 red. $1165 : 5 = 233$. So, there are 233 green balloons and $4 \cdot 233 = 932$ red balloons.
2. $\frac{1}{5}$ part of all balloons will be green balloons. $4 + 1 = 5$, and $\frac{4}{5}$ will be red. $\frac{1}{5} \cdot 1165 == 233$

Also, we can say that the ratio of green balloons to all balloons is $\frac{1}{5}$, or 1 to 5, in other words, one out of 5 balloons is green (and therefore 4 are red)

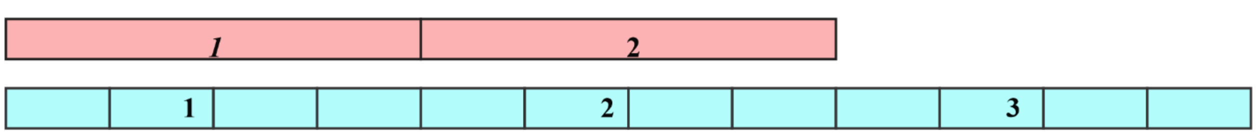
Problem2:

Irene has a total of 1686 red, blue and green balloons for sale. The ratio of the number of red balloons to the number of blue balloons was 2:3. After Irene sold 3/4 of the blue balloons, 1/2 of the green balloons and none of the red balloons, she has 922 balloons left. How many blue balloons did Irene have at first?

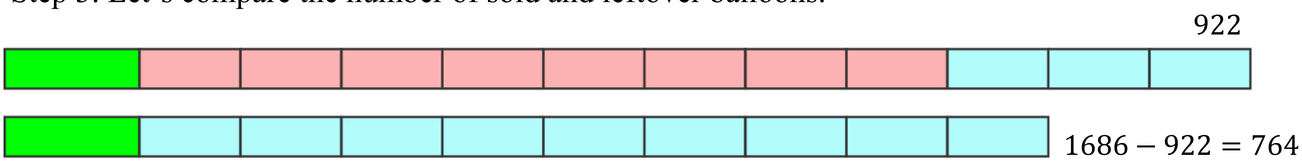
Step 1. For each 2 red balloons there are three blue balloons, so we can show all red and blue balloons as:



We took as "unit" a half of the red balloons. The number of blue balloons is $\frac{3}{2}$ times more than number of red balloons (or three times as much as a half of the red ones)



Step 2. $\frac{3}{4}$ of the blue balloons were sold. We can't divide 3 "units" into 4 parts, without getting fractions. So, let's find LCM of 3 and 4 and divide the number of blue balloons into 12 parts.
Step 3. Let's compare the number of sold and leftover balloons.



Number of sold and unsold green balloons are the same, red balloons are all left, as well as $\frac{1}{4}$ of blue balloons. As we can see 2 small "units" of blue balloons are $922 - 764 = 158$, or one such "unit" is 79. Total amount of blue balloons is $158 \cdot 6 = 948$. The number of red balloons is

$$\frac{2}{3} \cdot 948 = 632.$$

Number of green ones is $1686 - (632 + 948) = 106$. Can we solve the problem by writing equations?

Let's try.

$G + B + R = 1686$

$3R = 2B$

$\frac{1}{2}G + R + \frac{1}{4}B = 922$

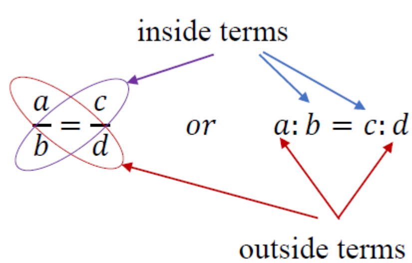
$\frac{1}{2}G + R + \frac{1}{4}B - \left(\frac{1}{2}G + \frac{3}{4}B\right) = 922 - 764$

$R - \frac{1}{2}B = 158$

$\frac{2}{3}B - \frac{1}{2}B = 158 \quad \Rightarrow \quad \left(\frac{4}{6} - \frac{3}{6}\right)B = 79 \quad \Rightarrow \quad B = 6 \cdot 158$

To cook a raspberry jam according to recipe I need to combine three cups of berries and 2 cups of sugar, or for each 3 cups of raspberries go 2 cups of sugar; ratio of raspberries and sugar (in volume) is $3:2$. If I bought 27 cups of raspberries, how many cups of sugar do I need to put to my jam?

$$\frac{3}{2} = \frac{27}{x}$$



Two ratios which are equal form a proportion.
Proportions have several interesting features.

1. The products of inside and outside terms are equal.

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c$$

It can be easily shown:

$$\frac{a}{b}=\frac{c}{d}\Leftrightarrow\frac{adb}{b}=\frac{cdb}{d}\Leftrightarrow ad=cb$$

2. Also, two inverse ratios are equal:

$$\frac{a}{b}=\frac{c}{d}\quad\Leftrightarrow\quad\frac{b}{a}=\frac{d}{c}$$

Indeed:

$$\frac{a}{b}=\frac{c}{d}\quad\Leftrightarrow\quad a\cdot d=b\cdot c\quad\Leftrightarrow\quad\frac{ad}{ac}=\frac{bc}{ac}\quad\Leftrightarrow\frac{d}{c}=\frac{b}{a}$$

3. Two outside terms can be switched:

$$\frac{a}{b}=\frac{c}{d}\quad\Leftrightarrow\quad\frac{d}{b}=\frac{c}{a}$$

$$\frac{a}{b}=\frac{c}{d}\quad\Leftrightarrow\quad a\cdot d=b\cdot c\quad\Leftrightarrow\quad\frac{ad}{ab}=\frac{bc}{ab}\quad\Leftrightarrow\frac{d}{c}=\frac{b}{a}$$

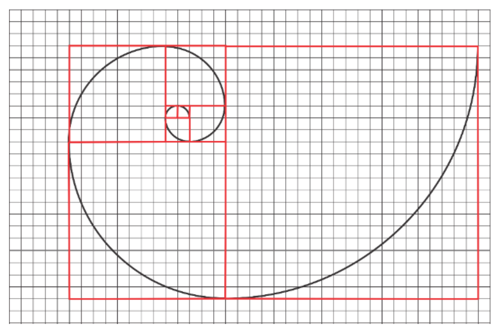
4. Two inside terms can be switched as well.

$$\frac{a}{b}=\frac{c}{d}\quad\Leftrightarrow\quad\frac{a}{c}=\frac{b}{d}$$

Famous ratios.

Let's measure the circumference and the diameter of a circle.

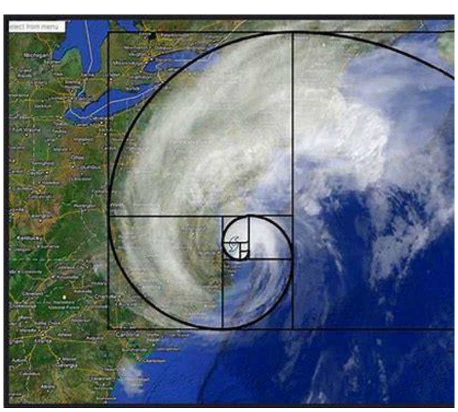
$$\frac{l}{d}=\pi$$

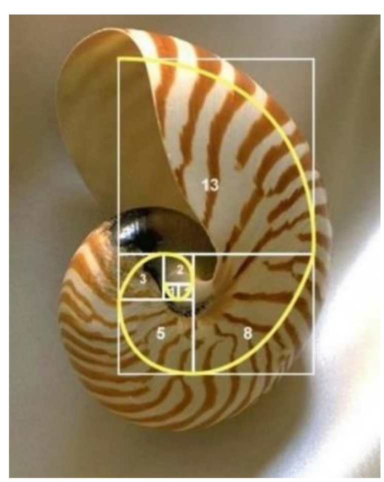


Fibonacci sequence:

$$1,1,2,3,5,8\ldots..$$

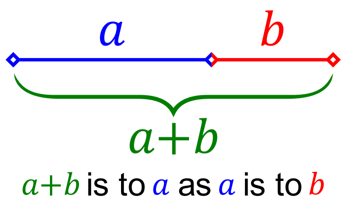
$$F_n=F_{n-1}+F_{n-2}$$

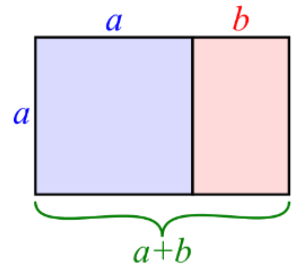




Golden ratio

$$\frac{a+b}{a} = \frac{a}{b} \cong 1.618$$





Ratio of two consecutive Fibonacci numbers becoming closer and closer to golden ratio when numbers increase.

Terms of Fibonacci Sequence	
F0 = 0	F10 = 45
F1 = 1	F11 = 89
F2 = 1	F12 = 134
F3 = 2	F13 = 233
F4 = 3	F14 = 377
F5 = 5	F15 = 610
F6 = 8	F16 = 987
F7 = 13	F17 = 1597
F8 = 21	F18 = 2584

Terms of Fibonacci Sequence	
F9 = 34	F19 = 4181

$\frac{8}{5} = 1.6;\quad \frac{13}{8} = 1.625;\quad \frac{21}{13} = 1.615\ldots;\quad \frac{34}{21} = 1.619\ldots$

Exercises :

1. Five classes of mathematics a day, is it a lot? Five classes of mathematics a year, is it a lot?

2. If we increase the weight of an ant by 1g, will it be a significant increase (average weight of an ant is less than 10mg)? If we increase the weight of an elephant by 1 g?

   Come up with your own examples when increasing the value by the same amount can give a completely different qualitative result.

3. Mr. Robinson was paid $590 for a job that required 40 hours of work. At this rate, how much should he be paid for a job requiring 60 hours of work?

4. If two pounds of meat will serve 5 people, how many pounds will be needed to serve 13 people?

5. 6 oxen or 8 cows can graze a field in 28 days. How long would 9 oxen and 2 cows take to graze the same field?

6. Bronze is an alloy of tin and copper. (Tin and copper are metals; they are melted together to get an alloy which is called bronze). How much copper and how much tin are there in the 80 kg piece of bronze, if the ratio of tin to copper in bronze is 3 to 17?

7. Fresh watermelon weighted 10 kg and contained 99% of water. In the store the watermelon lost some amount of water and now contains only 98% of water. What is its weight now?

8. Dry cranberries contain 25% of water. How much water should be evaporated from 5 kg of fresh cranberries to get dry cranberries, if fresh cranberries contain 85% of water?

9. To do her homework, Julia solved math problems, wrote an essay, and did a history project. It took her 2 hours and 15 minutes to finish all the assignments. The ratios of the times she spends doing math, writing the essay, and doing history project are 3:2:1. How much time did she spend for each of her subjects?

10. A merchant accidentally mixed candies of the first type (priced at $3 per pound) with candies of the second type (priced at $2 per pound). At what price should this mixture be sold to obtain the same total amount, given that it is known that initially the total cost of all candies of the first type was equal to the total cost of all candies of the second type?

11. Johnson's family had a breakfast. Each member of the family drunk one full cap of coffee, with or without milk. Robert had $\frac{1}{4}$ of all milk and $\frac{1}{6}$ of all coffee. How many members of the Johnson's family had a breakfest?