MATH 5: HANDOUT 23
GEOMETRY 3.

Congruence tests for triangles

Recall that by definition, to check that two two triangles are congruent, we need to check that corresponding angles are equal and corresponding sides are equal; thus, we need to check 6 equalities. However, it turns out that in fact, we can do with fewer checks.

Axiom 1 (SSS Rule)**.** *If* $AB = A'B'$, $BC = B'C'$ *and* $AC = A'C'$ *then* $\triangle ABC \cong \triangle A'B'C'$.

We can also try other ways to define a triangle by three pieces of information, such as a side and 2 angles (ASA), three angles (AAA), or two sides and an angle. For the two sides and an angle, there are two versions, one in which the two sides are adjacent to the given angle (SAS) and the other in which one of the given sides is opposite to the given angle (SSA). It turns out that ASA and SAS do indeed define a triangle:

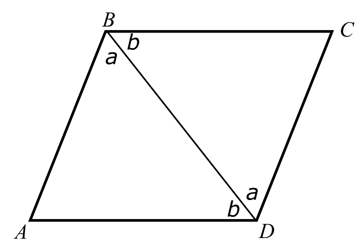
Axiom 2 (Angle-Side-Angle Rule)**.** *If* $\angle A = \angle A'$, $\angle B = \angle B'$ *and* $AB = A'B'$*, then* $\triangle ABC \cong \triangle A'B'C'$.

This rule is commonly referred to as ASA rule.

Axiom 3 (SAS Rule)**.** *If* $AB = A'B'$, $AC = A'C'$ *and* $\angle A = \angle A'$*, then* $\triangle ABC \cong \triangle A'B'C'$.

These rules — and congruent triangles in general — are very useful for proving various properties of geometric figures. As an illustration, we prove the following useful result.

Theorem. *Let* $ABCD$ *be a parallelogram. Then* $AB = CD$, $BC = AD$*, i.e. the opposite sides are equal.*



Proof. Let us draw diagonal BD. Then the two angles labeled by letter a in the figure are equal as alternate interior angles (because $AB \parallel DC$); also, two angles labeled by letter b are also equal. Thus, triangles $\triangle ABD$ and $\triangle CDB$ have a common side BD and the two angles adjacent to it are the same. Thus, by ASA, these two triangles are congruent, so $AD = BC$, $AB = CD$. $\square$