Math 4. Class Work 23

Exponent 2

Exponentiation is a mathematical operation written as a^n

$$a^n = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times}$$

We have two numbers, the **base *a*,** and the **exponent *n*.** The exponent indicates how many copies of the base are multiplied together.

- When ***n* is a positive** integer, we have repeated multiplication of the base n times. In that case, a^n is called the ***n*-th power of *a***, or *a* raised to the power *n*.

Properties of exponent:

1. $a^n = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times}$
2. $a^n \cdot a^m = a^{n+m}$
3. $(a^n)^m = a^{n \cdot m}$
4. $a^1 = a$, for any a
5. $a^0 = 1$, for any $a \neq 0$
6. $(a \cdot b)^n = a^n \cdot b^n$

- A positive number raised to any power will result in a positive number.
- A negative number, raised to a power, represented by an even number, is positive, represented by an odd number is negative.

$a^1 = a$ and $a^0 = 1$ for any number *a,* but 0.

$$a^{10} = a^{10} \cdot 1 = a^{10+0} = a^{10} \cdot a^0$$

Last time added: If a number a in a power n is divided by the same number in a power m,

$$\frac{a^n}{a^m} = \frac{\underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}}{\underbrace{a \cdot a \cdot \ldots \cdot a}_{m\ times}} = \left(\underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}\right) : \left(\underbrace{a \cdot a \cdot \ldots \cdot a}_{m\ times}\right) = a^n : a^m = a^{n-m}$$

Division $\frac{a^5}{a^2} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a} = a^{5-2} = a^3$

Problems

1. Write the following expressions in a shorter way, replacing product with power:

Examples:

$$(-a)\cdot(-a)\cdot(-a)\cdot(-a)=(-a)^4, \qquad 3m\cdot m\cdot m\cdot 2k\cdot k\cdot k\cdot k=6m^3k^4$$

$a)$ $(-y)\cdot(-y)\cdot(-y)\cdot(-y);$

$b)$ $(-5m)(-5m)\cdot 2n\cdot 2n\cdot 2n;$

$c)$ $-y\cdot y\cdot y\cdot y;$

$d)$ $-5m\cdot m\cdot 2n\cdot n\cdot n;$

$e)$ $(ab)\cdot(ab)\cdot(ab)\cdot(ab)\cdot(ab)\cdot(ab);$

$f)$ $p-q\cdot q\cdot q\cdot q\cdot q;$

$g)$ $a\cdot b\cdot b\cdot b\cdot b\cdot b;$

$h)$ $(p-q)\cdot(p-q)\cdot(p-q);$

2. Simplify the expressions:

$a)$ $2^4+2^4;$ $b)$ $2^m+2^m;$ $c)$ $2^m\cdot 2^m;$

$d)$ $3^2+3^2+3^2;$ $e)$ $3^k+3^k+3^k;$ $f)$ $3^k\cdot 3^k\cdot 3^k;$

3. What will be last digit of

$a)$ $2^{22};$ $b)$ $3^{33};$ $c)$ $4^{44};$ $d)$ $5^{55};$ $e)$ $6^{66};$ $f)$ $7^{77};$

4. Compare:

Example: What is greater 31^{11} or 17^{14}?

We can see that $31<32=2^5;\ 2^4=16<17,$

$$31^{11}<32^{11}\ , \quad then\ (2^5)^{11}=2^{55}$$

$$(17)^{14}>16^{14}\ ,\ then\ (2^4)^{14}=2^{56}$$

We can write the following:

$$31^{11}<32^{11}=2^{55}<2^{56}=16^{14}<(17)^{14}$$

$$31^{11}<17^{14}$$

a) 127^{23} and 513^{18}

b) 9997^{10} and 100003^8

c) 5^{300} and 3^{500}

5. Exponents can help write numbers in their expanded form. In our base-10 place value system, the position of the digit indicates which power of 10 should be multiplied by the digit. Write the numbers in expanded form.

   a) **Last class:** $345 = 300 + 40 + 5 = 100 \cdot 3 + 10 \cdot 4 + 1 \cdot 5 = 10^2 \cdot 3 + 10^1 \cdot 4 + 10^0 \cdot 5$

   Very large numbers can be written using this system, as well as very small numbers.

   $$0.3 = \frac{1}{10} \cdot 3 = 10^{-1} \cdot 3;$$

   $$0.456 = \frac{1}{10} \cdot 4 + \frac{1}{100} \cdot 5 + \frac{1}{1000} \cdot 6 = 10^{-1} \cdot 4 + 10^{-2} \cdot 5 + 10^{-3} \cdot 6$$

   b) $0.1 =$

   c) $0.01 =$

   d) $0.021 =$

   e) $0.4361 =$

   f) $0.10201 =$

   g) $0.654321 =$

6. Reduce the fractions

   $a)\ \frac{49^4 \cdot 7^5}{7^{12}};$ $\quad b)\ \frac{3^{10} \cdot 27}{81^3};$ $\quad c)\ \frac{125^3 \cdot 5^7}{5^{18}};$