Math 4. Class Work 25

Combinatorics 2

Factorial ! = a short notation representing the multiplication of all whole numbers from 1 to the last number before the ! sign

$$3! = 1 \cdot 2 \cdot 3 = 6$$
$$n! = 1 \cdot 2 \cdot 2 \cdot 4 \cdot \dots \cdot (n-1) \cdot n$$

Combinations: in how many ways can we arrange or group elements of different types.

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This can be done by counting all possible combinations. We must be careful if the order of the elements in the new groups matters; if the order doesn't matter, we may overestimate the number of calculations.

Problems

1. If

$$1! = 1 \cdot 1$$

$$2! = 1 \cdot 2$$

$$3! = 1 \cdot 2 \cdot 3 = 3 \cdot 2!$$

$$4! = 1 \cdot 2 \cdot 3 \cdot 4 = 4 \cdot 3!$$

How much is 0!

2. Calculate the product

a) 3! and $\frac{3!}{2}$ b) $\frac{3!}{4!}$ and $\frac{3!}{5!} \times 4$

- 3. How many two-digit numbers can be composed from the digits 1, 2, and 3 without repetition of digits?
- 4. If repetition is allowed, how many two-digit numbers can be composed from the digits 1, 2, and 3?
- 5. How many three-digit numbers can be composed of the digits 1,2 and 3 without and with repetition?

- 6. A mother has 2 apples and 3 pears. Each day of the week, she gives one fruit to her kid for lunch. How many different orders are there to give these fruits? Hint: what if the apples and pears were different colors? What if you can not tell the apples (pears) apart? We will overcount the apples by a number 2! the number of ways to arrange two apples. Similarly, we will overcount the pears.
- Peter took 5 exams at the end of the year. The possible grades for exams are A, B, C, and D. How many different ways are there to fill out his report card?
- 8. There are red and green pencils in a box. How many pencils do you have to take out of the box without seeing them to be sure that you have at least 2 pencils of the same color?
- 9. If there are 5 different colors of pencils in a box, how many pencils do you have to take out to be sure that you have at least 2 of the same color? 3 of the same color?
- $10. \ {\rm Open} \ {\rm the} \ {\rm parenthesis} \ {\rm and} \ {\rm simplify}$

a) $3 \cdot (x + 1) + x \cdot (x + 1) =$ b) $x \cdot (2x + 3) + 3 \cdot (2x + 3) =$

11. Open the parenthesis and simplify

a)
$$(x + 1)(x + 2) =$$

b) $(x + 3) \cdot (2x + 5) =$

