Math 4. Class Work 24

Combinatorics

Factorial ! = a short notation representing the multiplication of all whole numbers from 1 to the last number before the ! sign

$$3! \ = \ 1 \cdot 2 \cdot 3 = 6$$
$$n! \ = \ 1 \cdot 2 \cdot 2 \cdot 4 \cdot ... \cdot (n-1) \cdot n$$

Combinations: in how many ways can we arrange or group elements of different types.
This can be done by counting all possible combinations. We must be careful if the order of the elements in the new groups matters; if the order doesn't matter, we may overestimate the number of calculations.

Problems

1. John had 5 friends at his birthday party.
   To the first friend, he gave 1/6 of the cake.
   To the second, he gave 1/5 of the remaining cake.
   To the third, he gave 1/4 of the new remainder.
   To the fourth, he gave 1/3 of what was left at that moment.
   The last piece, John split in half with the fifth friend.
   Who got the largest piece?

2. There are 5 chairs and 5 kids in the room. In how many ways can the kids sit on these chairs?

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What are the similarities between the two problems? Can you see the difference between them?

3. How many different ways are there to choose 3 students to participate in the math Olympiad, essay competition, and history competition out of 8 students (one student in each competition)?

4. How many different ways are there to choose a team of 3 students out of 8 to participate in the math Olympiad?

5. Simplify the following fractions: $\dfrac{5!}{7!}$; $\quad\quad \dfrac{9!}{7!}$ $\quad\quad \dfrac{n!}{(n-2)!}$;

6. Reduce the fractions

$a)$ $\dfrac{49^4 \cdot 7^5}{7^{12}}$; $\quad\quad$ $b)$ $\dfrac{3^{10} \cdot 27}{81^3}$; $\quad\quad$ $c)$ $\dfrac{125^3 \cdot 5^7}{5^{18}}$;

7. How many different ways are there to put 64 books on the shelf?

8. There are 20 students in the 4th grade math team.
    They have to choose 4 participants to go to the county math Olympiad. How many ways are there to choose these 4 students from the team of 20?

9. In the restaurant, there are 3 choices of starters, 4 choices of entrees, and 5 choices of tasty desserts in the fixed-price dinner menu. How many different ways are there to fix a dinner for the restaurant's clients?

10. How many two-digit numbers can be composed from the digits 1, 2, and 3 without repetition of digits?

11. How many two-digit numbers can be composed from the digits 1, 2, and 3 if repetition is allowed?

12. How many three-digit numbers can be composed of the digits 1,2 and 3 without and with repetition?