$a.\ \frac{32}{65} \dots \frac{49}{65};$ $b.\frac{7}{96} \dots \frac{7}{12};$ $c.\ \frac{14}{23} \dots \frac{14}{37};$ $d.\ \frac{18}{19} \dots \frac{16}{19};$

29. Find the unknown:

$a.\ x + \frac{5}{36} = \frac{13}{36};$ $b.\ y - \frac{16}{49} = \frac{27}{49};$

$c.\ \frac{8}{21} + k = \frac{17}{21};$ $d.\ \frac{48}{56} - t = \frac{39}{56};$

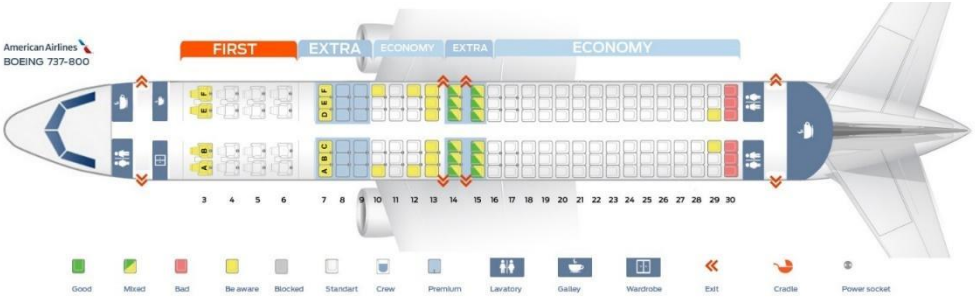
30. To which natural numbers the following fractions are equal to?

Example: $\frac{12}{3} = 4$

$a.\ \frac{16}{8};$ $b.\ \frac{18}{2};$ $c.\ \frac{24}{6};$ $d.\ \frac{30}{3};$ $e.\ \frac{35}{35};$ $f.\ \frac{51}{17}$

Chapter 8.

Coordinates.



Coordinates are a set of values that show an exact position. How many values do we need to show the exact position of a point on a number line? How many values do we need to find our place in a theater? In a plane? What we can use as values?

For example, the Johns family lives in Big Village, on Main Street, house number 33, NY, USA. To describe the location of Johns' house, we use several pieces of information, such as:
Country: USA, State: NY, village: Big, Street: Main, House: 33.
On a number line, each point represents a number, and each number is linked to a point if an origin (the point at 0), a unit segment, and the positive direction are defined or can be defined based on the known information. This number is the coordinate of a point on the line in the defined system. The absolute value of this number tells us how many unit segments lie between this point and the origin, while the sign indicates on which side of the origin this point is located.

Example 1.

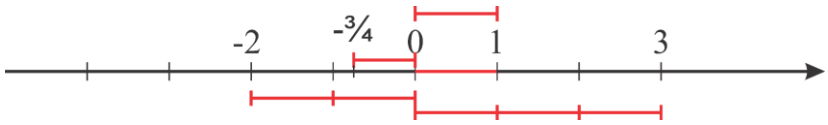
Find the coordinates of points A, B, C, D, E, F, G, and H on the number line below:

To find the coordinate of a point, we need to find how many unit segments can fit into the distance between 0 and the point, and then apply the corresponding sign, either plus or minus.
On this number line there are two known coordinates, -2 and 2. Exactly 4 unit segments fit between two points, with 0 positioned exactly in the middle. The coordinate of point E is 0, E(0). The coordinate of the point F is 1 and $\frac{1}{4}$, F$\left(1\frac{1}{4}\right)$.
Each number has a property "absolute value", it shows how far this number is from 0, the origin of the coordinate line. The formal definition of absolute value of any number is:

$$\begin{cases} |a| = a, & if\ a \geq 0 \\ |a| = -a, & if\ a < 0 \end{cases}$$

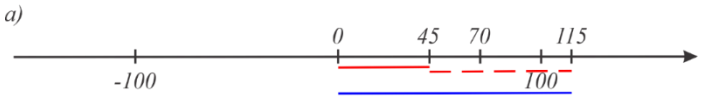
Absolute value of 2 is 2, absolute value of (-2) is also 2,

$$|2| = 2; \qquad |-2| = 2$$

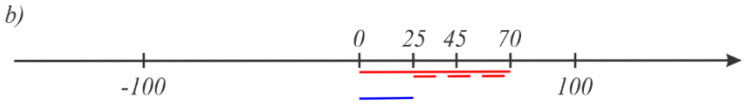


The addition and subtraction of positive and negative numbers along the coordinate axis.

Two segments represent the absolute values of numbers 45 and 70. When we need to add 45 and 70, we can sum their absolute values and the result will be positive. Addition of two positive numbers produces the positive result.
The blue segment represents the absolute value of the result.

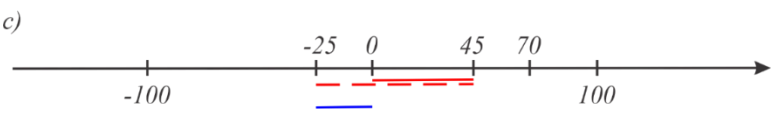


$$45 + 70 = 70 + 45 = 70 - (-45) = 45 - (-70) = 115$$



The result of subtraction of 45 from 70 is 25. The blue line represents the absolute value of the resalt, segment with the length equal to the difference of two other segments.

$$70 - 45 = 70 + (-45) = 25$$



The result of the subtraction of 70 from 45 will be the opposite of the result of $70 - 45$. The absolute value of the result will be the same (the difference between 70 and 45) but the sign will be " $-$ "

$$45 - 70 = 45 + (-70) = -(|70| - |45|) = -25$$

So, if you need to subtract one number from the other: subtract the smaller number from the greater number and decide about the resulting sign.
Example 1:

$$25 - 77;$$

25 is smaller than 77, so the answer should be negative, the absolute value of the difference is the same as the absolute value of the difference of $77 - 25$; in other words, the result is a number, opposite to the difference $77 - 25$.

$$25 - 77 = -(77 - 25) = -(+52) = -52$$

Example 2:

$$168 - 230;$$

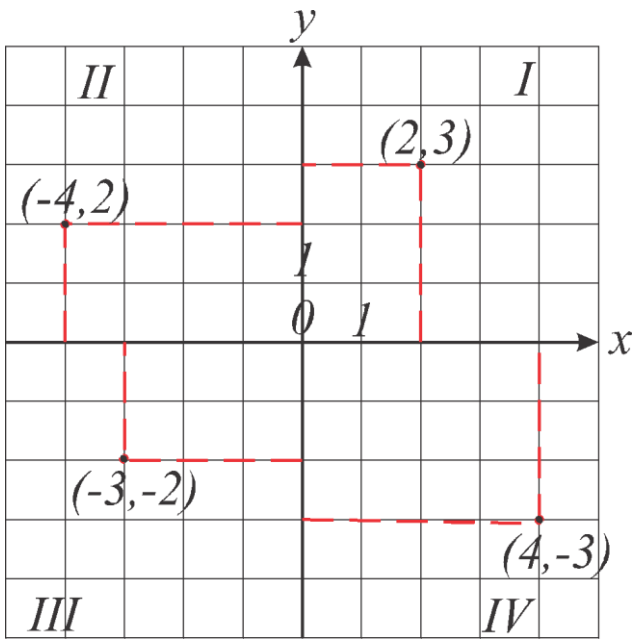
$168 < 230$, we can find the difference $230 - 168 = 62$ and write " $-$ " .

$$168 - 230 = -(230 - 168) = -62$$

Let's take a look on the equation like

$$|x| = 12$$

To which number x should be equal to make this equation true?
We are looking for a number which absolute value is 12. There are two such numbers, 12 and -12. On a plane, each point corresponds to a unique ordered pair of numbers. To define these pairs, 2 perpendicular number lines are usually used. These two number lines intersect at the point called origin, associated with pair (0,0), have the same unit segment, and are called axis, usually x and y axis.

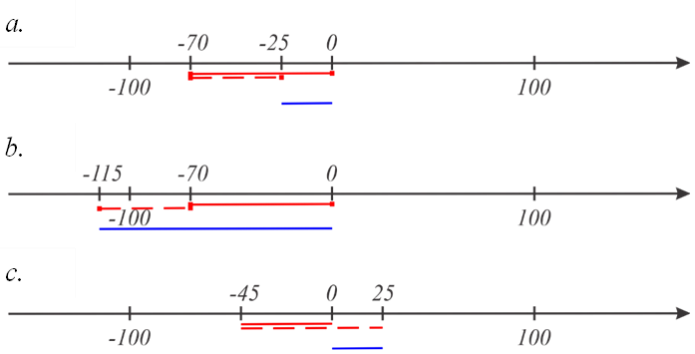


In this particular coordinate system, the two numbers allied with each point of the plane describe the distance from the point to both axes, and the signs of these numbers represent a quadrant where the point lies (quadrants I, II, III, and IV in the image above). Such a pair of numbers is an ordered pair, so the pair (n, m) and the pair (m, n) are linked to 2 different points. The absolute value of the first number in the pair is the distance to the y axis. The absolute value of the second one is the distance to the x axis.

Exercises:

1. Mark the points A(0), B(1), C$\left(-1\frac{1}{2}\right)$, D(5), E$(-5)$, F$(-3)$, G(3) on the number line in your notebook.

2. Write a problem, represented by the drawing:



3. Evaluate: Example: $a.\ \ 7 + (-2) = 5$

$a.\ (-10) + (+11);$ $b.(-7) + (-6);$ $c.\ (-4) + (+2);$

$d.\ (-12) + (+3);$ $e.15 - 18;$ $f.\ (-11) + (-20);$

$g.\ 20 - 21;$ $h.(-100) + (-150);$ $i.\ (-3) + (+4);$

4. Evaluate:

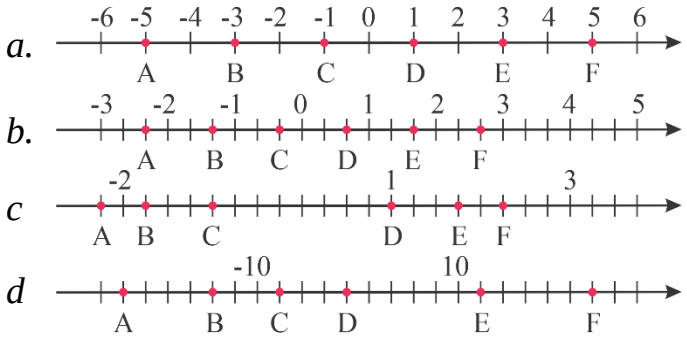
$a.\ 18 - (-5);$ $b.\ -21 - (-20);$ $c.\ 46 - (-6);$ $d.-30 - (-30);$

$e.\ 15 - (-20);$ $f.\ -50 - (-5);$ $g.\ 23 - (-28);$ $h.-31 - (-62);$

5. Fill the table:

a	20	-15	30	-10	-7
b	7	8	-9	-6	9
$a - b$					
$b - a$					

6. Write the coordinates of the points A, B, C, D, E, and F for the axes below:

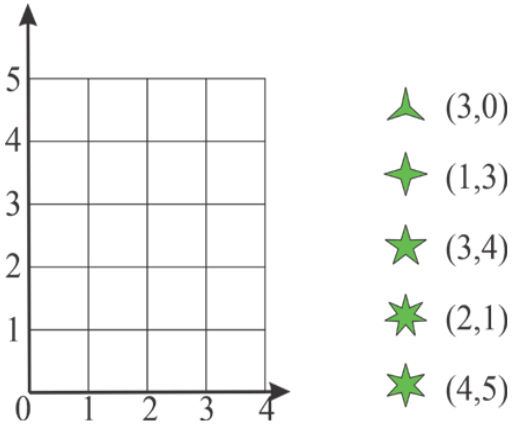


7. On the axis points $A(-2)$ and B(3) marked. Mark the origin (point is coordinate equal to 0) and unit segment. Find the coordinates of the points C, D, E.

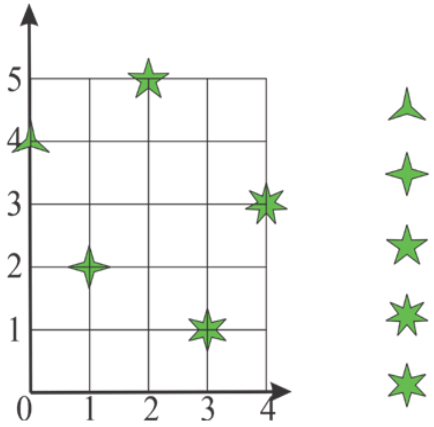
8. In your notebook, on the number line mark the points which absolute value is
   a. |3|; *b.* |5|; *c.* |7|; *d.* |0|.

9. Draw the pictures in your notebook.

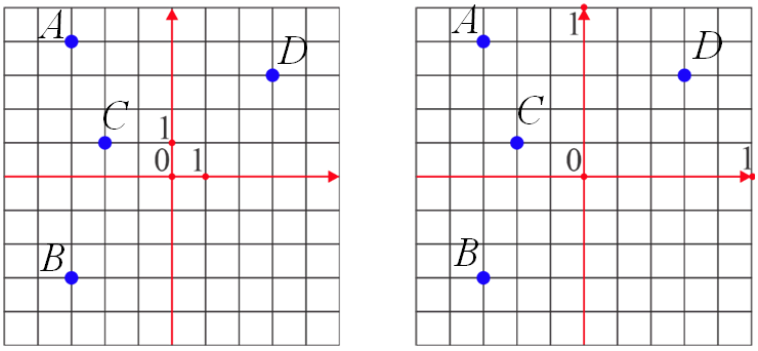
Draw stars with corresponding coordinates:



Write coordinates of the stars:



10. Find coordinates of points A, B, C, D in two different coordinate systems:



11. Draw a coordinate system (only positive quadrant I). Create a picture by coordinates (connect points in the order):

    C_1 (2, 0), C_2 (2, 10), C_3 (4, 12), C_4 (12, 12), C_5 (18, 14), C_6 (18, 16), C_7 (20, 14), C_8 (22, 14), C_9 (24, 12), C_{10} (24, 14), C_{11} (25, 12), C_{12} (26, 12), C_{13} (26, 14), C_{14} (28, 12), C_{15} (28, 10), C_{16} (24, 8), C_{17} (22, 8), C_{18} (18, 6), C_{19} (18, 0), C_{20} (14, 0), C_{21} (14, 4), C_{22} (6, 4), C_{23} (6, 0), C_1(2, 0).

12. Write coordinates of the points A1, A2, …, A15 from the picture:

13. It's known that $x \cdot y = 12$. What is the value of the expressions:

$a.\ x \cdot (y \cdot 5);$ $b.\ (x \cdot 2) \cdot y;$ $c.\ y \cdot (x \cdot 10);$ $d.\ (y \cdot 2) \cdot (x \cdot 3);$

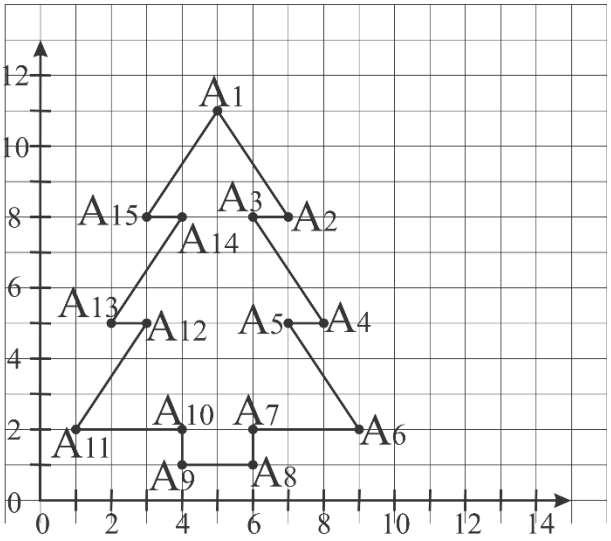
14. Evaluate by the convenient way:

Example:

$$25 \cdot 24 = 25 \cdot (4 \cdot 6) = (25 \cdot 4) \cdot 6 = 100 \cdot 6 = 600$$

$$75 \cdot 8 = (25 \cdot 3) \cdot (4 \cdot 2) = (25 \cdot 4) \cdot (3 \cdot 2) = 100 \cdot 6 = 600$$

$a.\ 36 \cdot 25;$ $b.\ 25 \cdot 12;$ $c.\ 75 \cdot 24;$ $d.\ 150 \cdot 42;$



15. Explain the way the calculations were performed in the following examples. Using the algorithm, do the calculations:

Examples:

$$238 \cdot 6 = (200 + 30 + 8) \cdot 6 = 200 \cdot 6 + 30 \cdot 6 + 8 \cdot 6 = 1200 + 180 + 48 = 1428$$

$$97 \cdot 14 = (100 - 3) \cdot 14 = 100 \cdot 14 - 3 \cdot 14 = 1400 - 42 = 1358$$

$a.\ 104 \cdot 14;$ $b.\ 102 \cdot 22;$ $c.\ 98 \cdot 3;$ $d.\ 196 \cdot 15;$

16. Evaluate by the most convenient way:

Example:

$$29 \cdot 25 + 15 \cdot 6 + 19 \cdot 15 = 29 \cdot 25 + 15 \cdot (6 + 19) = 29 \cdot 25 + 15 \cdot 25 = 25 \cdot (29 + 15) = 25 \cdot 44 = 25 \cdot 4 \cdot 11 = 100 \cdot 11 = 1100$$

$a.\ 12 \cdot 17 + 35 \cdot 13 + 17 \cdot 23;$ $b.\ 41 \cdot 80 - 25 \cdot 41 + 55 \cdot 29;$

$c.\ 26 \cdot 18 + 26 \cdot 17 + 14 \cdot 35$

17. In the number 38*6107*, replace the asterisks with digits so that the resulting number is a multiple of

a. multiple of 2;
b. multiple of 5;
c. multiple of 10;
d. multiple of 3;
e. multiple of 9;
f. multiple of 2 and 3;
g. multiple of 15;
h. multiple of 45.

18. Mary calculated that if she gives each of her guests 2 cookies, 4 cookies will remain. However, if she gives each guest 3 cookies, 2 cookies will be missing. How many guests did Mary invite?

19. Evaluate:

$$a.\ \frac{\frac{1}{2}\cdot\frac{2}{3}}{\frac{1}{3}};\qquad b.\ \frac{\frac{5}{8}}{\frac{3}{4}\cdot\frac{1}{2}};\qquad c.\ \frac{1}{\frac{1}{2}\cdot\frac{1}{6}};\qquad d.\ \frac{\frac{2}{5}\cdot\frac{3}{4}}{15}$$

20. Evaluate (answer is 4):

$$\left(4\frac{7}{9}-2\frac{5}{6}\right):1\frac{5}{9}+\frac{4}{9}\cdot 6\frac{3}{16}$$

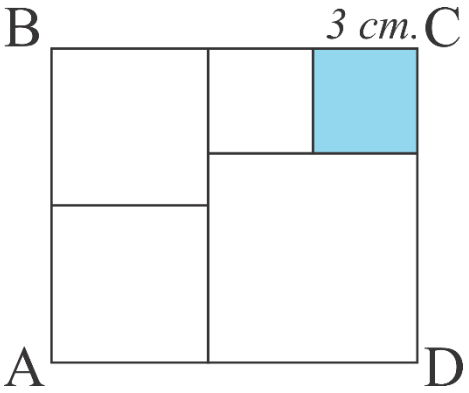
21. A student borrowed an interesting book from the library. It has 75 pages. On the first day, he read $\frac{3}{5}$ of the entire book, and on the second day, he read $\frac{2}{5}$ of the remaining pages. How many pages does he have left to read?

22. Which sign $(+, -, \cdot, \div)$ should be placed instead of * to make the following equalities true statements.

$$a.\ \frac{7}{8} * 1\frac{1}{7} = 1\qquad b.\ 2 * 1\frac{1}{3} = \frac{2}{3};\qquad c.\ \frac{3}{7} * \frac{4}{7} = \frac{3}{4};\qquad d.\ \frac{3}{10} * \frac{5}{6} = \frac{1}{4};$$

23. Fill the empty spaces in the magic squares with digits from 2 to 8 (from 0 to 8 for the full square) so that all the columns, rows, and diagonals have the same sum of numbers.

	0	
1		



24. The rectangle ABCD divided into 5 squares, the side of the shaded square is 3 cm. Find the sides of the rectangle.

25. Solve the following riddle (each letter represents a digit)

```
  EAT
+ THAT
-------
 APPLE
```

26. A boy has as many brothers as sisters, but his sister has twice less sisters than brothers. How many girls and how many boys are in this family?

27. Evaluate (try to do it in a convenient way):

$$101101 \cdot 555 - 101 \cdot 555555;$$

28. In the cinema, there are two viewing halls: the red hall and the blue hall. The red hall has 40 rows, with 45 seats in each row. The blue hall has 25 rows, with 24 seats in each row. How many times more seats are there in the red hall compared to the blue hall?

29.