

Work problems (combine labor problems).

Mary can eat her birthday cake in 10 minutes, and Peter can eat the same cake in 15 minutes. How fast will they eat the same cake together?

These kinds of problems are related to the amount of work done per unit of time; we can call it “rate”. To solve the problem, we have to find out what part of the cake Mary will eat in 1 minute. If she can eat the whole cake in 10 minutes, she only eats $\frac{1}{10}$ of the cake in one minute. Peter will eat $\frac{1}{15}$ of the cake in 1 minute. If they will start eating the cake simultaneously, each minute

$$\frac{1}{10} + \frac{1}{15} = \frac{3}{30} + \frac{2}{30} = \frac{5}{30} = \frac{1}{6}$$

will be eaten. We don’t know, how many minutes are needed, but the rate with which the cake will be disappearing is $\frac{1}{6}$ per minute:

$$x(\text{minutes}) \cdot \frac{1}{6} (\text{part of the cake}) = 1(\text{whole cake})$$

So, they will need exactly

$$x = 1(\text{whole cake}) : \frac{1}{6} (\text{parts}) = 1 \cdot 6 = 6 \text{ minutes}$$

The bathtub fills with cold water in 6 minutes 40 seconds, with hot water in 8 minutes. Additionally, if the plug is removed from the full bathtub, the water will drain out in 13 minutes 20 seconds. How much time will it take to fill the bathtub completely, assuming both taps are open but the bathtub is not plugged?

This problem is very similar to a previous one. If only cold-water tap is open, it will take 6 min. and 40 second $\left(6\frac{2}{3} = \frac{20}{3}\right)$ to fill the tub. In one minute $\frac{3}{20}$ of the tub

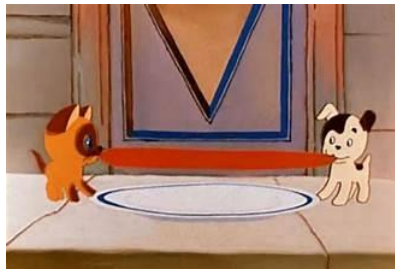
will be filled. If only hot water tap is open $\frac{1}{8}$ of the tab will be filled in one minute.

Also, $\frac{3}{40}$ of the tab will be drained out each minute, if the plug is not installed.

$$\frac{3}{20} + \frac{1}{8} - \frac{3}{40} = \frac{6}{40} + \frac{5}{40} - \frac{3}{40} = \frac{8}{40} = \frac{1}{5}$$
$$t \cdot \frac{1}{5} = 1; t = 5 \text{ minutes.}$$

Exercises:

1. The distance between two villages is covered by a pedestrian in 60 minutes and by a cyclist in 20 minutes. How many minutes will it take for them to meet if they set off simultaneously towards each other from these villages?
2. A cat can eat the sausage in 10 minutes, a dog can eat it in 2.5 minutes. How fast will they eat the sausage together?



3. Mary, Peter, and Julia are going to do the spring cleanup in their garden. Mary can do the job in 4 hours, Peter can do the full clean up in 3 hours, Julia need 6 hours to do the job. How fast they will do it together?
4. A swimming pool can be filled by pump A in 3 hours and by pump B in 6 hours, each pump working on its own. At 9 am pump A is started. At what time will the swimming pool be filled if pump B is started at 10 am?
5. One tractor can plow a field in 15 days, while another can do it in 24 days. Which tractor will plow more: the first one in 8 days or the second one in 11 days?

6. The older brother can clean up the room in 2 hours, the younger brother can completely ruin it in 3 hours. In how many hours will the room be cleaned if they are locked together in the messy room? (it's a math problem, the answer "they will play games" will not be accepted!)

7. To bake 100 pancakes, Mom needs 30 minutes, while Dad needs 40 minutes. Son can eat 100 pancakes in an hour. Mom and Dad continuously make pancakes without stopping, while Son continuously eats them. After how much time from the beginning of this process will there be exactly 100 pancakes on the table?

