

MATH 10
ASSIGNMENT 1: A BRIEF REVIEW
SEPTEMBER 15, 2024

WELCOME TO THE NEW YEAR AT SCHOOLNOVA!!

And welcome to Math 10! It is an exciting course, full of new material which will be very useful in your mathematical education. The main topics we will study are linear algebra and real analysis, which are the two of the main mathematical tools which scientists (of all sciences), engineers, etc. use. Not only they are useful, they revolutionized mathematics itself.

One of the main news about Math 10 is that it is completely proof-based. We will also learn to do some calculations (like inverting matrices, for example), but the focus will be on proving general facts, like proving why such method of calculation works. This is quite a challenging change of perspective from simply solving problems, but it is an important step in one's mathematical education. It will help to grasp the real meaning and purpose of the calculations we do.

Besides linear algebra and real analysis, there are a few additional topics which might be covered depending on the interest of the students and on the time we have available. Here is a rough plan:

- Overview of past materials. Complex numbers.
- Linear algebra: vectors, vector operations, systems of linear equations, matrices, matrix inverses.
- Real analysis: metric spaces, properties of real numbers, limits of sequences, series.
- Notions of abstract algebra: Groups, Lagrange's theorem, vector spaces and algebras
- (Possibly) Introduction to topology: metric spaces, topological spaces, surfaces and triangulations, problems of coloring maps.

We will try to do much of the homework in class so that you do not need to spend too much time on it at home. As usual, all HW assignments and other information will be posted online at <http://www.schoolnova.org>.

You should bring your HW to class next week and hand it to the teacher; I will grade it and return to you. Don't worry if you cannot do all of the problems, but always attempt all of them and write down your progress even if you couldn't complete a proof. Please write neatly and in a separate sheet of paper.

We also plan to participate in math competitions, and you should register soon. Check your SchoolNova email for the link to register.

If you have any questions, please contact me by email: kirillov@schoolnova.org.

PREVIOUS MATERIAL

A lot of the material you covered last year will be useful for us, especially:

Complex numbers.

Mathematical induction.

Sets and functions.

Planar geometry: vectors, dot product.

Today we will review these topics; no new material is given.

HOMEWORK

1. Compute

$$\frac{1}{2 - 3i}$$

2. Let $z = 1 + i\sqrt{3}$.

- (a) Compute z^{2022} .
- (b) Compute $1 + z + z^2 + \dots + z^{29}$.

3. The polynomial $x^3 - 39x + 70$ has three roots. Two of these roots are $x_1 = 2$, $x_2 = 5$. What is the third root?
4. Prove that for any $n \geq 1$, we have

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n - 1)}{2}$$

5. Let $f: A \rightarrow B$, $g: B \rightarrow C$ be injective functions. Prove that then the composition $g \circ f: A \rightarrow C$ is injective.
(Recall that a function $f: A \rightarrow B$ is injective (one-to-one) if for any $y \in B$, the equation $f(x) = y$ has at most one solution. Equivalently, f is **not** injective if there exist $x_1, x_2 \in A$ such that $x_1 \neq x_2$, but $f(x_1) = f(x_2)$.)
6. A subset S of the real line is called *bounded* if one can find an interval $[-M, M]$ which contains all elements of S . Can you write this definition using no words but only quantifiers, variables, arithmetic operations and inequalities?
7. Consider a parallelogram $ABCD$ with vertices $A(0, 0)$, $B(3, 6)$, $D(5, -2)$. Find the coordinates of:
- vertex C
 - midpoint of segment BD
 - Midpoint of segment AC