46th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior O-Level Paper, Fall 2024

(The result is computed from the three problems with the highest scores.)

points	problems
4	1. Consider a circumscribed pentagon $ABCDE$. Its incenter lies on the diagonal AC. Prove that $$AB + BC > CD + DE + EA.$$ *Egor Bakaev*
4	2. Pete puts 100 stones in a row: black one, white one, black one, white one, ..., black one, white one. In a single move either Pete chooses two black stones with only white stones between them, and repaints all these white stones in black, or Pete chooses two white stones with only black stones between them, and repaints all these black stones in white. Can Pete with a sequence of moves described above obtain a row of 50 black stones followed by 50 white stones? *Egor Bakaev*
4	3. A positive integer M has been represented as a product of primes. Each of these primes is increased by 1. The product N of the new multipliers is divisible by M. Prove that if we represent N as a product of primes and increase each of them by 1 then the product of the new multipliers will be divisible by N. *Alexandr Gribalko*
5	4. A mother and her son are playing. At first, the son divides a 300g wheel of cheese into 4 slices. Then the mother divides 280g of butter between two plates. At last, the son puts the cheese slices on those plates. The son wins if on each plate the amount of cheese is not less than the amount of butter (otherwise the mother wins). Who of them can win irrespective of the opponent's actions? *Alexandr Shapovalov*
5	5. The set consists of equal three-cell corners (L-triminoes), the middle cells of which are marked with paint. A rectangular board has been covered with these triminoes in a single layer so that all triminoes were entirely on the board. Then the triminoes were removed leaving the paint marks where the marked cells were. Is it always possible to know the location of the triminoes on the board using only those paint marks? *Alexandr Gribalko*

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points problems

3 1. Pete puts 100 stones in a row: black one, white one, black one, white one, ..., black one, white one. In a single move either Pete chooses two black stones with only white stones between them, and repaints all these white stones in black, or Pete chooses two white stones with only black stones between them, and repaints all these black stones in white. Can Pete with a sequence of moves described above obtain a row of 50 black stones followed by 50 white stones?

Egor Bakaev

4 2. Two polynomials with real coefficients have the leading coefficients equal to 1. Each polynomial has an odd degree that is equal to the number of its distinct real roots. The product of the values of the first polynomial at the roots of the second polynomial is equal to 2024. Find the product of the values of the second polynomial at the roots of the first one.

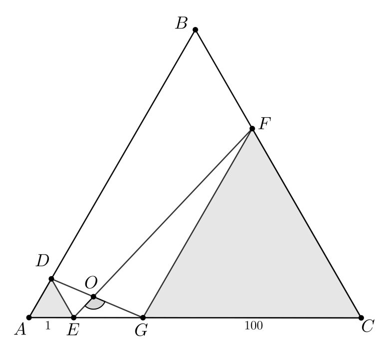
Sergey Yanzhinov

5 3. There are five positive integers written in a row. Each one except for the first one is the minimal positive integer that is not a divisor of the previous one. Can all these five numbers be distinct?

Boris Frenkin

5 4. In an equilateral triangle ABC the segments ED and GF are drawn to obtain two equilateral triangles ADE and GFC with sides 1 and 100 (points E and G are on the side AC). The segments EF and DG meet at point O so that the angle EOG is equal to $120°$. What is the length of the side of the triangle ABC?

Mikhail Evdokimov



5 5. There is a balance without weights and there are two piles of stones of unknown masses, 10 stones in each pile. One is allowed an unlimited number of weighing iterations, but only 9 stones at most fit on any plate of the balance. Is it always possible to determine which stone pile is heavier or establish that they are equal?

Sergey Dorichenko