Homework 14

Length contraction

We discuss another striking effect of the special relativity – the *length contraction*. During last class we discussed time dilation. We found that if we force the speed of light to be the same in all inertial reference frames, we have to accept the fact that time flows slower for the object which is moving with respect to us. It is important to avoid confusion: the moving clock are ticking slower compared with the clock at rest, but work "normally" for the person who is moving with the clock. For the person who is moving with the clock, the clock is at rest and the time it measures is called *proper time*.

From the point of view of the person who is moving with the clock, the clock at rest is lagging, since in the moving reference frame tis clock are moving. It is extremely difficult to accept that the same clock can move faster or slower depending on whether the observer is moving or stays still with respect to the clock. It seems to be highly nontrivial, almost miraculous. But this is how our world works. Time is relative and depends on our state of motion. If we were able to move close to the speed of light we definitely could adapt the effect of the special relativity as our everyday experience and nontriviality of this theory could be less striking. Today we will discuss another parameter which the observers in different inertial frames do not agree on. This is length.

Let us consider our train carriage passing by a tree at a speed V. There are two observers. One is traveling in the carriage; the other is staying on the ground. For the moving observer, the carriage is at rest, but the tree is moving back at a speed V. The observer in the carriage measures the time interval Δt' between the moments of the tree passing the front and back of the carriage. This time interval can be expressed as:

$$\Delta t' = \frac{L}{V} \qquad (1)$$

Here L is the length of the carriage. The observer at the ground measures the same time interval:

$$\Delta t = \frac{L}{V} \qquad (2)$$

But, according to our previous result:

$$\Delta t' = \Delta t\sqrt{1 - \frac{V^2}{c^2}} \qquad (3),$$

So combining expressions (1) – (3), we obtain something strange:

$$\frac{L}{V} = \frac{L}{V}\sqrt{1 - \frac{V^2}{c^2}} \qquad (4), \text{ or}$$

$$L = L\sqrt{1 - \frac{V^2}{c^2}} \qquad (5)$$

Expession (5) is apparently wrong. But we can correct of if we assume that the length of the object which is moving with respect to us is not the same of the object at rest. Let us call the length of the moving object as *L'*. Then, expression (5) can be rewritten as:

$$L = L'\sqrt{1 - \frac{V^2}{c^2}} \qquad (6),$$

or

$$L' = \frac{L}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (7).$$

The object which is moving with respect to us becomes shorter in the direction of motion. The dimensions, perpendicular to the direction of motion stay intact. This effect is known as ***the length contraction.***

Let try to see what happens in the Rossi-Hall experiment (described in the previous homework) from "the muon point of view", or, speaking more scientifically, in the muon reference frame. In this reference frame, the muon is at rest but the Earth and its atmosphere are moving towards the muon at a speed of 0.995c. The time of muon in this reference frame is the proper time and the lifetime is 2.2ns. But the moving atmosphere experiences length contraction, so the distance to the Earth surface (the atmosphere thickness) is contracted almost 10 times! So in this case as well a lot of muons are able to reach the Earth surface.

Problems:

1. A spaceship is 100 meters long at rest. If it travels at 0.8c (c -the speed of light) to an observer at Earth, what is its length measured from Earth?
2. A rod is moving along its length. The length of the moving rod measured by the observer at rest is halve of its proper length. How fast is the rod moving?