Homework 23

Black body radiation

During last class we discussed black body radiation. We learned that any object of nonzero temperature has to emit electromagnetic radiation and top absorb electromagnetic radiation emitted by its surrounding. The object which is hotter that its surrounding emits more than absorbs and cools down. A cold object obtains more energy from surroundings that the object emits and warms up.

We learned that the amount of energy emitted from an object depends on the object material and on its temperature. The higher object's temperature the more energy emitted from the surface per unit time. It turns out that the total power ***R(T)*** emitted from a unit surface area of absolutely black body (the body which is able to emit and absorb electromagnetic radiation of all frequencies) obeys *Stefan-Boltzmann law*:

$$R(T) = \sigma \cdot T^4 \qquad (1)$$

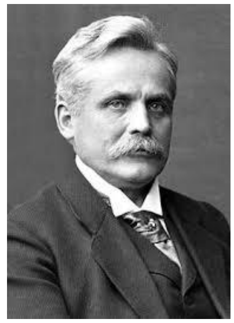
Here $\sigma = 5.67 \cdot 10^{-8}\ W/(K \cdot m^2)$ is the *Stefan-Boltzmann constant*. As you may remember, another name for R(T) is ”radiant exitance”. The law is named after Austrian-Slovenian physicist, mathematician and poet Joseph Stefan (1835-1893) and Austrian physicist and philosopher Ludwig Boltzmann (1844-1906).

Joseph Stefan   Ludwig Boltzmann

As the temperature of a black body increases, not only radiant exitance but the spectrum of electromagnetic radiation is changing. The wavelength of maximum emission decreases with increasing temperature so that:

$$w = \lambda_{max} \cdot T = 2898\ [\mu m \cdot K] \qquad (2)$$

Expression (2) is called the *Wien displacement law* and w is the *Wien constant*. It is named after a German physicist Wilhelm Wien (1864-1928).



Wilhelm Wien

Black body radiation spectrum.

The spectral radiant exitance of a black body can be expressed as:

$$R(\lambda,T)=\frac{2\pi hc^2}{\lambda^5}\cdot\frac{1}{e^{\frac{hc}{\lambda kT}}-1} \qquad (3)$$

It looks a bit too complicated, but just for a first glance. Here ***h*** is the Planck's constant, **c** is the speed of light, $\boldsymbol{\lambda}$ is the wavelength, ***T*** is the temperature in absolute units (Kelvin scale), ***k*** is the Boltzmann constant ($k = 1.38\cdot 10^{-23} J/K$) and ***e*** is mathematical constant , often referred as the Euler's number or Napier's number ($e \approx 2{,}718281828\ldots$). As the famous $\boldsymbol{\pi}$ number, ***e*** is irrational so there is no period in the mantissa of ***e***.

What does the expression (3) mean? It shows the energy per unit wavelength, emitted from unit area of a black body at temperature T in a narrow wavelength range near the wavelength $\boldsymbol{\lambda}$. It shows the contribution of different wavelengths to the total power, emitted by a black body. The plots of the spectral radiant exitance of a black body at different temperatures are shown in Figure 1.

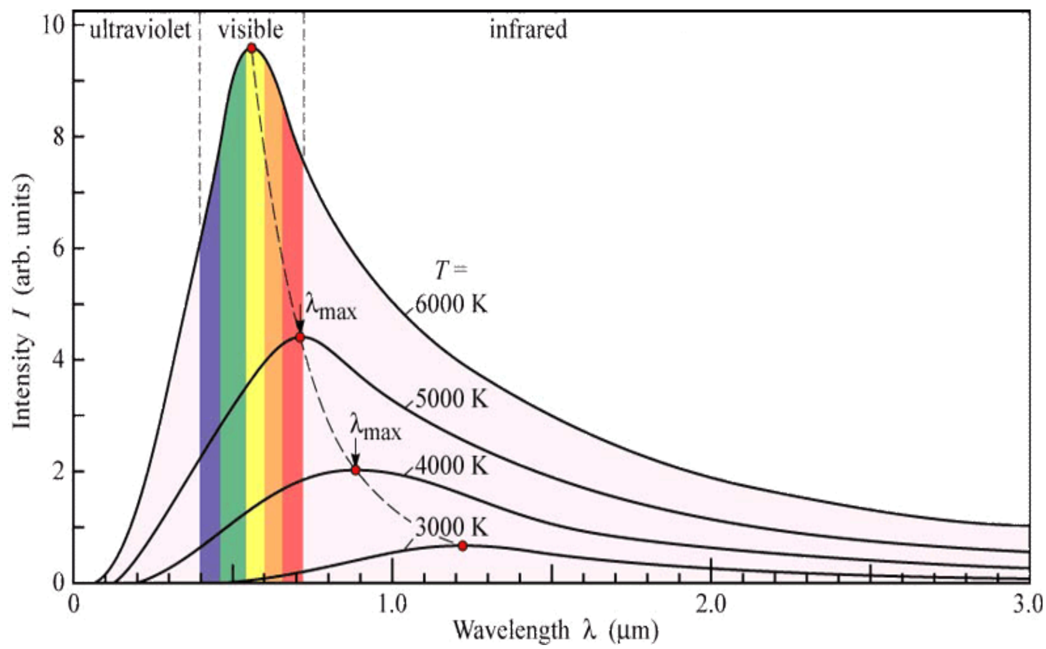


Figure 1. Schematic plots of the expression (1) at different temperatures. (the image is taken from http://www.globalchange.umich.edu/globalchange1/current/lectures/universe/universe.html)

Problems:

1. Assume that the Earth is in thermal equilibrium and radiates energy into space at the same rate at which it receives it from the Sun. At what orbit radius around the Sun would the ocean freeze? How does the answer compare with the actual Earts's orbit radius of $1.5 \cdot 10^{11}\ m$? (Let us assume that the Earth behaves as a black body, the Sun's radiant flux is $3.9 \cdot 10^{26}\ W$ and there is fresh water in the ocean ☺)

2. Find units of $R(\lambda, T)$ and explain the result.

3. Please simplify the expression (1) for the case of $\frac{hc}{\lambda kT} \ll 1$. For this one can use the following mathematical approximation: if $x \to 0$, then $e^x \approx 1 + x$.