Homework 15

Interval

We learned that observers in different inertial frames (the frames moving with respect to each other at a constant velocities) do not agree with each other about length, time, momentum and kinetic energy. Il looks like the only parameter, as we know for now, they are all agree on is the speed of light. In class we learned then there is another parameter - ***interval***, which is the same in all inertial reference frames.

Let us consider two events which are separated by a distance Δx and time period Δt in a certain reference frame. Then quantity $c\Delta t$, where c is the speed of light, gives the distance the light passes for the time Δt. The magnitude l calculated as:

$$l^2 \equiv c^2(\Delta t)^2 - (\Delta x)^2 \qquad (1)$$

is called ***interval.*** Let us try to use another reference frame which is moving with respect to the first one at a constant velocity V. In this new reference of frame the two events will be separated by other time period $\Delta t'$ and distance $\Delta x'$ and the interval l' will be calculated as:

$$(l')^2 = c^2(\Delta t')^2 - (\Delta x')^2 \qquad (2)$$

Let us compare the intervals between two events calculated in two different inertial reference frames. For this we will use Lorentz transformations:

$$\Delta x' = \frac{\Delta x - V\Delta t}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (3)$$

$$\Delta t' = \frac{\Delta t - \frac{V\Delta x}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (4)$$

If we plug $\Delta t'$ and $\Delta x'$ given by (3) and (4) into expression (2) we will have:

$$(l')^2 = c^2(\Delta t')^2 - (\Delta x')^2 = \frac{1}{\left(1-\frac{V^2}{c^2}\right)}\left[c^2\left(\Delta t - \frac{V\Delta x}{c^2}\right)^2 - (\Delta x - V\Delta t)^2\right] =$$

$$\frac{1}{\left(1-\frac{V^2}{c^2}\right)}\left[c^2\Delta t^2 - 2V\Delta t\Delta x + \frac{V^2(\Delta x)^2}{c^2} - (\Delta x)^2 + 2V\Delta t\Delta x + V^2(\Delta t)^2\right] =$$

$$\frac{1}{\left(1-\frac{V^2}{c^2}\right)}\left[c^2\Delta t^2\left(1 - \frac{V^2}{c^2}\right) - (\Delta x)^2\left(1 - \frac{V^2}{c^2}\right)\right] = c^2(\Delta t)^2 - (\Delta x)^2 = l \quad (5)$$

So we can see that the interval stays the same in all inertial reference frames.

If $l^2 < 0$, the interval is imaginary (in mathematical meaning of this word) and the distance between two events is more than the distance which the light could pass during the time period between the events. We know that no information can propagate faster than light. It means that there could not be causal link between two events, in other words, one event cannot be a reason for the other. Imaginary intervals are called "space-like intervals". If the interval between two events is space-like than there is an inertial reference of frame in which the two events happen in the same time.

If $l^2 > 0$, the interval is real and the distance between two events is less than the distance which the light could pass during the time period between the events. In this case one event can cause the other. Real intervals are called "time-like intervals". If the interval between two events is time-like than there is an inertial reference of frame in which the two events happen in the same place.

Problems:

1. Two events are separated by distance ***Δx*** and time interval ***Δt*** in a certain reference frame. The interval between two events is time-like. Find the time period between the events in the reverence of frame where the events happen in the same place.

2. In a certain frame of reference event A happens 200m away from the origin in $5 \cdot 10^{-6}$s after the clock is started. Event B happens at the line connecting the location of event A and the origin but 1000 m farther away from the origin than the event A. The time of event B is $2 \cdot 10^{-6}$after the clock is started. Is the interval between two events lime-like or space-like?