MOMENTUM AND MOMENTUM CONSERVATION

JANUARY 8, 2024

Theory recap

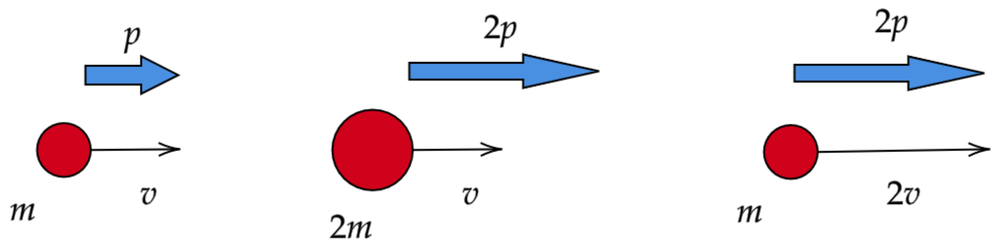
Imagine that a car and a big truck are moving with the same speed. Which one of them is harder to stop? One way to answer this question is from the point of view of Newton's second law. The truck has larger mass, so to supply it with the same acceleration as the car, one needs to apply larger force.

There is another reasoning leading to the same conclusion. It is related to Newton's second law but formulated differently. We introduce a new quantity called momentum which is equal to product of mass and velocity:

$$\vec{p} = m\vec{v}$$

We see that for larger mass and for larger speed momentum gets larger. A truck has larger momentum than a car, so it is harder to stop the truck than the car.

As we see from the definition of momentum, it is a vector. It has magnitude equal to product of mass and speed and its direction is the same as direction of velocity. This is illustrated on a figure below.



Units of momentum are the product of units of mass and speed: $\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}} = \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}}$. Unlike force, there is no special name for this unit.

Since momentum is a product of mass and velocity, if we change either mass or velocity of an object, its' momentum will change. Moving objects with changing mass are somewhat exotic, so we will not consider it and will focus on change in velocity. Change in velocity requires acceleration. By Newton's second law acceleration requires a force. So, change in momentum requires some force. Note that we do not say anything new here, it is Newton's first law expressed in different terms. If no net force is applied to a body, its momentum stays the same.

How momentum changes when there is a force will be discussed next time. Today we will look at momentum of a system of bodies instead. Say we have two blocks, one with mass m_1 and velocity $\vec{v}_1$ and the other with mass m_2 and velocity $\vec{v}_2$. We can find their total momentum by adding their individual momenta:

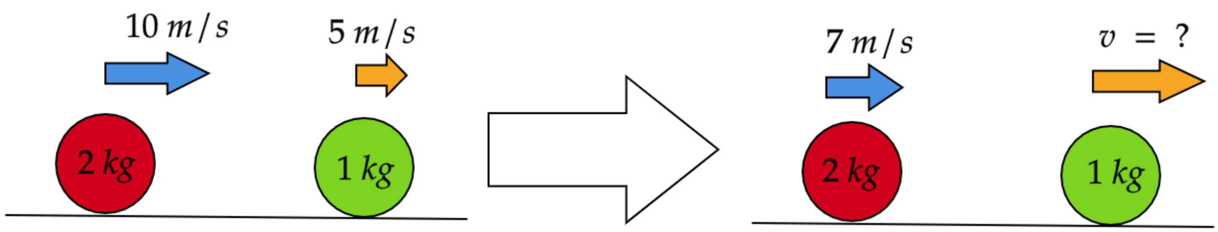
$$\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

Remember that for one body if there is no net force, its momentum stays constant. There is a generalization of this to our system: if there is no net external force, the total momentum of a system stays constant.

First, let us understand what external force means. We have two bodies in our system. They interact with each other and with some external objects (the Earth, floor, etc.). Forces of interaction between the objects in the system are called internal. They *always* come in pairs of equal and opposite forces: if m_1 acts on m_2 with force $\vec{F}$ then by Newton's third law m_2 acts on m_1 with force $-\vec{F}$. Therefore, when we consider the whole system and add up all the forces, these internal forces *always* cancel out. On the other hand, forces between the objects in the system and external objects, called external forces, do not cancel out. So only external forces could change properties of the whole system, such as total momentum. If there is no net external force, momentum does not change.

The statement that some quantity does not change (stays constant in time, is conserved) is called a conservation law. We have formulated **momentum conservation law**: total momentum is conserved if there is no net external force. It is a very-very important law. There are many cases when using it is much simpler than using Newton's laws directly. Moreover, though we will not encounter this in our class, momentum conservation law still stays applicable even when Newton's aren't applicable anymore - like in relativity theory and in quantum theory.

Let us apply momentum conservation law to study a collision of two balls on a horizontal table without friction. The balls have masses $m_1 = 2$ kg and $m_2 = 1$ kg. Before the collision they respectively had velocities $v_1 = 10$ m/s and $v_2 = 5$ m/s directed to the right. After the collision the m_1 ball has velocity $v_1' = 7$ m/s directed to the right. Let us find the velocity of the m_2 ball after the collision.



The main idea is that since there is no friction, there is no net external force (in the vertical direction gravity is balanced by the normal force). Therefore total momentum is conserved. Let us write total momentum before the collision choosing positive direction to the right:

$$p_{tot} = p_1 + p_2 = m_1 v_1 + m_2 v_2 = 25 \text{ kg} \cdot \frac{\text{m}}{\text{s}}$$

After the collision the total momentum is expressed via new velocities:

$$p_{tot} = p_1' + p_2' = m_1 v_1' + m_2 v = 14 + v$$

By setting one p_{tot} equal to the other we get

$$25 = 14 + v \implies v = 11 \text{ m/s}$$

Homework

1. A fox is chasing a small rabbit. The momentum of the fox is equal to the momentum of the rabbit. Will the fox catch the rabbit?

2. A 80 kg jogger runs with a constant acceleration of 0.2 m/s^2 for 10 seconds. How his momentum changed during this time?

3. A 10 kg ball moving on a horizontal plane at a speed of 10 m/s hits a 5 kg ball which was at rest before the collision. After the collision the smaller ball starts moving at a speed 10 m/s. Find the velocity of the heavy ball after the collision. Neglect friction.

***4.** An astronaut of mass 100 kg approaches a cosmic ship of mass 50000 kg by pulling a cable attached to the ship. Initial distance between the astronaut and the ship is 100 m and they both are initially at rest. What distance will the astronaut and the ship have traveled by their meeting time? Mass of the cable is negligible. *Hint: How are velocities of the astronaut and the ship related at every moment of time?*