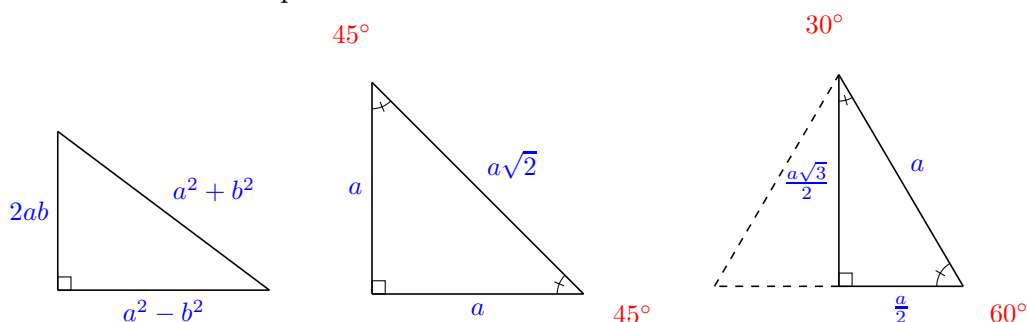


**MATH 7: HANDOUT 5**  
**RIGHT TRIANGLES AND PYTHAGOREAN THEOREM**

PYTHAGOREAN THEOREM

In a right triangle with legs  $a$  and  $b$ , and hypotenuse  $c$ , the square of the hypotenuse is the sum of squares of each leg.  $c^2 = a^2 + b^2$ . The converse is also true, if the three sides of a triangle satisfy  $a^2 + b^2 = c^2$ , then the triangle is a right triangle. Some Pythagorean triples are: (3,4,5), (5,12,13), (7,24,25), (8,15,17), (9,40,41), (11,60,61), (20,21,29).

To generate such Pythagorean triples, choose two positive integers  $a$  and  $b$ . Then plug the values into the sides as shown on the first picture:



Try to figure out why the sides of this triangle satisfy the Pythagoras' Theorem!

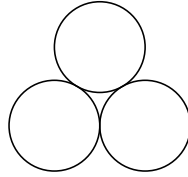
**45-45-90 Triangle:** If one of the angles in a right triangle is  $45^\circ$ , the other angle is also  $45^\circ$ , and two of its legs are equal. If the length of a leg is  $a$ , the hypotenuse is  $a\sqrt{2}$ .

**30-60-90 Triangle:** If one of the angles in a right triangle is  $30^\circ$ , the other angle is  $60^\circ$ . Such triangle is a half of the equilateral triangle. That means that if the hypotenuse is equal to  $a$ , its smaller leg is equal to the half of the hypotenuse, i.e.  $\frac{a}{2}$ . Then we can find the other leg from the Pythagoras' Theorem, and it will be equal to  $\frac{a\sqrt{3}}{2}$ .

Homework problems on back

### HOMEWORK

1. Come up with 5 different triplets of numbers that satisfy Pythagorean theorem.
2. What is the altitude and area of an equilateral triangle of side length 4?
3. What is the area and diagonal length of a square with side length 4?
4. A regular hexagon is inscribed in a circle of radius 6. What is the area of the hexagon?
5. What is the diagonal of a cube of side length 2?
6. What is the height of a stack of 3 congruent circles of radius 3cm?



[Hint: Consider the equilateral triangle formed by connecting the center of the circles]

7. In a trapezoid ABCD with bases AD and BC,  $\angle A = 90^\circ$ , and  $\angle D = 45^\circ$ . It is also known that  $AB = 10$  cm, and  $AD = 3BC$ . Find the area of the trapezoid.
8. In a right triangle ABC, BC is the hypotenuse. Draw AD perpendicular to BC, where D is on BC. The length of BC=13, and AB=5. What is the length of AD?
9. What is the area of a regular hexagon whose side is 5cm?
10. Rationalize the denominator:

(a)  $\frac{1}{1 + \sqrt{5}}$

(b)  $\frac{1}{1 - 2\sqrt{3}}$

(c)  $\frac{1}{4\sqrt{3} + 1}$

(d)  $\frac{2}{2\sqrt{2} - 1}$