MATH 7: HANDOUT 17
COORDINATE GEOMETRY 1: REVIEW. LINES AND CIRCLES. BASIC TRANSFORMATIONS

1. Coordinate Geometry: Introduction

In this section of the course we are going to study coordinate geometry. The basic notion is the **coordinate plane** – a plane with a given fixed point, called the **origin**, as well as two perpendicular lines – **axes**, called the x**-axis** and the y**-axis**. x-axis is usually drawn horizontally, and y-axis — vertically. These two axes have a **scale** – “distance” from the origin.

The scales on the axes allow us to describe any point on the plane by its **coordinates**. To find coordinates of a point P, draw lines through P perpendicular to the x- and y-axes. These lines intersect the axes in points with coordinates x_0 and y_0. Then the point P has x-coordinate x_0, and y-coordinate y_0, and the notation for that is: $P(x_0, y_0)$.

The **midpoint** M of a segment AB with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ has coordinates:

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

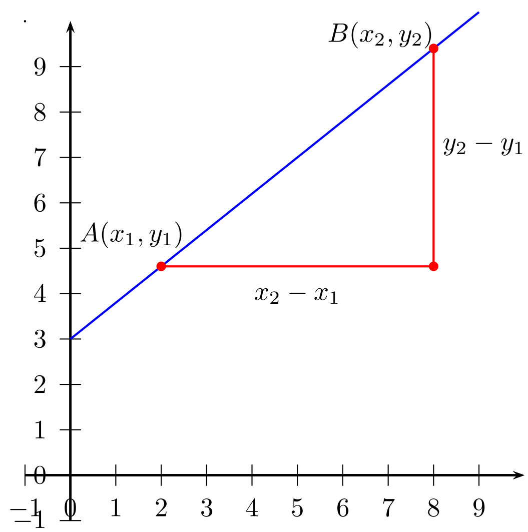
2. Lines

Given some relation which involves variables x, y (such as $x+2y=0$ or $y=x^2+1$), we can plot on the coordinate plane all points $M(x,y)$ whose coordinates satisfy this equation. Of course, there will be infinitely many such points; however, they usually fill some smooth line or curve. This curve is called the **graph** of the given relation.

Every relation (**equation**) of the form:

$$y = mx + b$$

where m, b are some numbers, defines a straight line. The slope of this line is determined by m: as you move along the line, y changes m times as fast as x, so if you increase x by 1, then y will increase by m:



In other words, given two points $A(x_1, y_1)$ and $B(x_2, y_2)$ **slope** can be computed by dividing change of y: $y_2 - y_1$ by the change of x: $x_2 - x_1$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Two non vertical lines are **parallel** if and only if they have the **same slope**.

In the equation $y = mx + b$, b is a y**-intercept**, and determines where the line intersects the vertical axis (y-axis).

The equation of the **vertical** line is $x = k$, and the equation of the **horizontal** line is $y = k$. Notice that in case of the vertical line, the slope is undefined.

3. Distance Between Points. Circles

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the following formula:

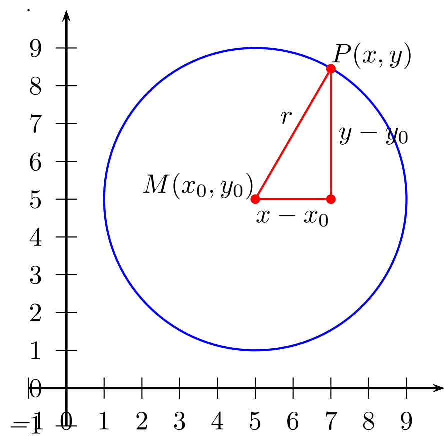
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This formula is a straightforward consequence of the Pythagoras' Theorem.

The equation of the circle with the center $M(x_0, y_0)$ and radius r is

$$(x - x_0)^2 + (y - y_0)^2 = r^2.$$

This equation means, that points (x, y) should be at distance r from the given point $M(x_0, y_0)$.

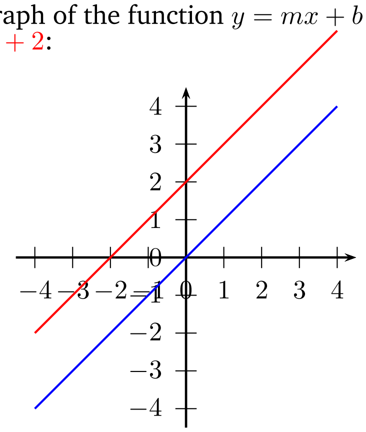


4. Graphs of functions

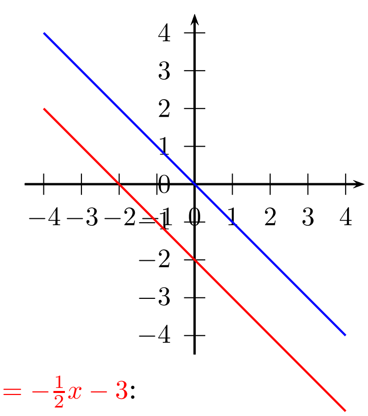
In general, the relation between x and y could be more complicated and could be given by some formula of the form $y = f(x)$, where f is some function of x (i.e., some formula which contains x). Then the set of all points whose coordinates satisfy this relation is called the **graph** of f.

Line. The graph of the function $y = mx + b$ is a straight line. The coefficient m is called the *slope*.

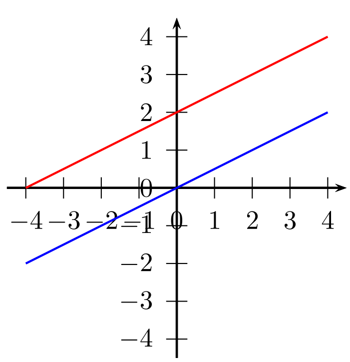
$y = x$; $y = x + 2$:



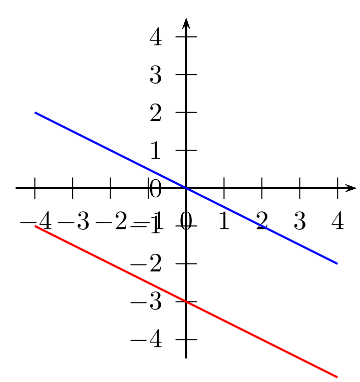
$y = -x$; $y = -x - 2$:



$y = \frac{1}{2}x$; $y = \frac{1}{2}x + 2$:

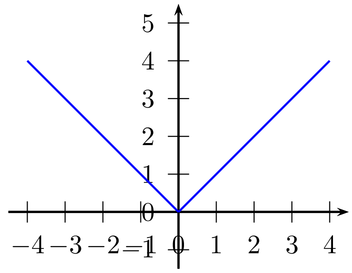


$y = -\frac{1}{2}x$; $y = -\frac{1}{2}x - 3$:



Graph of $y = |x|$

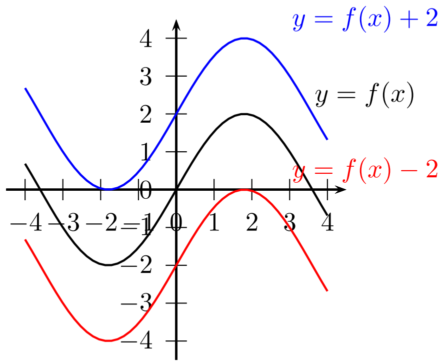
The figure below shows a graph of a function $y = |x|$.

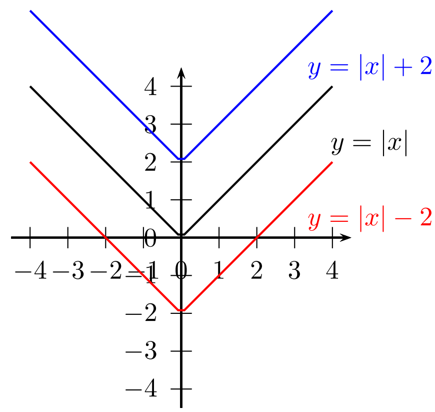


5. Trasnformations

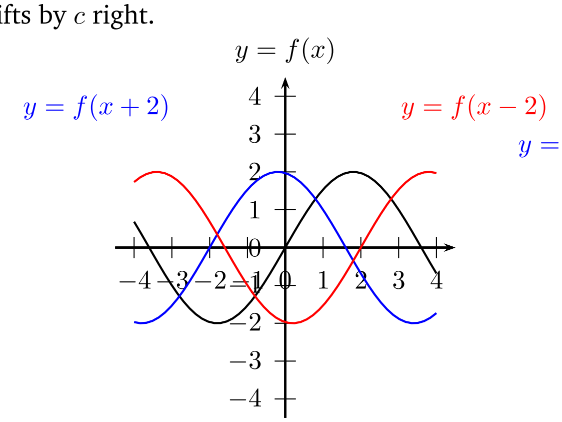
Having learned a number of basic graphs, we can produce new graphs, by doing certain transformations of the equations. Here are two of them.

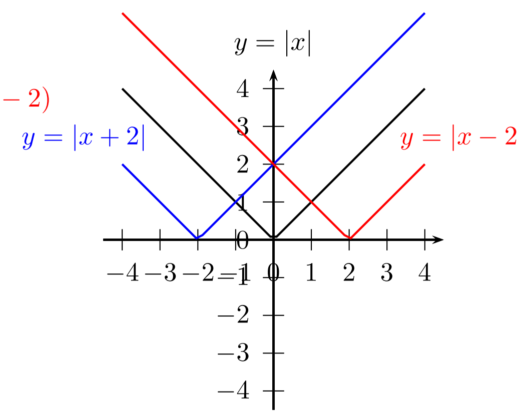
Vertical translations: Adding constant c to the right-hand side of equation shifts the graph by c units up (if c is positive; if c is negative, it shifts by $|c|$ down.)





Horizontal translations: Adding constant c to x shifts the graph by c units left if c is positive; if c is negative, it shifts by c right.





Homework

1. A point B is 5 units above and 2 units to the left of point $A(7,5)$. What are the coordinates of point B?
2. Find the coordinates of the midpoint of the segment AB, where $A=(3,11)$, $B=(7,5)$.
3. Draw points $A(4,1)$, $B(3,5)$, $C(-1,4)$. If you did everything correctly, you will get 3 vertices of a square. What are coordinates of the fourth vertex? What is the area of this square?
4. 3 points $(0,0)$, $(1,3)$, $(5,-2)$ are the three vertices of a parallelogram. What are the coordinates of the remaining vertex?
5. Consider the triangle ΔABC with the vertices $A(-2,-1)$, $B(2,0)$, $C(2,1)$. Find the coordinates of the midpoint of B and C. Find the length of the median (i.e. a median unites a vertex with the midpoint of the opposite side) from A in the triangle ΔABC.
6. What is the slope of a line whose equation is $y=2x$? What is the slope of a line whose equation is $y=mx$?
7. In this problem you will find equations that describe some lines.
   (a) What is the equation whose graph is the y-axis?
   (b) What is the equation of a line whose points all lie 5 units above the x-axis?
   (c) Is the graph of $y=x$ a line? Draw it.
   (d) Find the equation of a line that contains the points $(1,-1)$, $(2,-2)$, and $(3,-3)$.
8. For each of the equations below, draw the graph, then draw the perpendicular line (going through the point (0, 0)) and then write the equation of the perpendicular line

   (a) $y=2x$ (b) $y=3x$

   (c) $y=-x$ (d) $y=-\frac{1}{2}x$

   Can you determine the general rule: if the slope of a line is k, what is the slope of the perpendicular line?
9. Find the equation of the line through $(1,1)$ with slope 2.
10. Find the equation of the line through points $(1,1)$ and $(3,7)$. [Hint: what is the slope?]
11. (a) Find k if $(1,9)$ is on the graph of $y-2x=k$. Sketch the graph.
    (b) Find k if $(1,k)$ is on the graph of $5x+4y-1=0$. Sketch the graph.
12. Let l_1 be the graph of $y=x+1$, l_2 be the graph of $y=x-1$, m_1 be the graph of $y=-x+1$, and m_2 be the graph of $y=-x-1$.
    (a) Find the intersection point of l_1 and m_1; Label this point P and write down its coordinates.
    (b) Find the intersection point of l_2 and m_2; Label this point P and write down its coordinates.
    (c) Find the midpoint of AB and write down its coordinates.
    (d) Let C be the intersection point of l_1 with m_2, and D be the intersection point of l_2 with m_1. What kind of quadrilateral is $ABCD$?
    (e) Explain why l_1 and l_2 are parallel. What is the distance between them?
13. Find the intersection point of a line $y=x-3$ and a line $y=-2x+6$. Sketch the graphs of these lines.
14. Sketch graphs of the following functions:

    (a) $y=|x|+1$ (b) $y=|x+1|$ (c) $y=|x-5|+1$