

MATH 7
HANDOUT 11: PASCAL'S TRIANGLE CONTINUED

PASCAL'S TRIANGLE

Recall Pascal's triangle:

				1								
				1	1							
				1	2	1						
				1	3	3	1					
				1	4	6	4	1				
				1	5	10	10	5	1			
				1	6	15	20	15	6	1		
				1	7	21	35	35	21	7	1	
				1	8	28	56	70	56	28	8	1

Every entry in this triangle is obtained as the sum of two entries above it. The k -th entry in n -th line is denoted by $\binom{n}{k}$, or by ${}_nC_k$. Note that both n and k are counted from 0, not from 1: for example, $\binom{2}{1} = 2$.

These numbers appear in many problems:

- The number of paths (going only right and up) from a square on the chessboard to the square m units up and k units to the right is ${}_{m+k}C_k$. For example, there ${}_{14}C_7$ paths from lower left to upper right of the usual 8×8 chessboard.
- The number of words of length exactly n which can be written using only 2 letters, say U and R, and which contain exactly k U's.
- and any more.. see problems below!

PROBLEMS

1. Calculate $((5^{74} + 5^{77} - 5^{75}) \div 121 - 5^{73}) \div (5^{73} - 4 \cdot 5^{72})$
2. How many "words" of length 5 can one write using only the letters U and R, namely 3 U's and 2 R's? What if you have 5 U's and 3 R's? [Hint: each such "word" can describe a path on the chessboard, U for up and R for right...]
3. How many sequences of 0 and 1 of length 10 are there? sequences of length 10 containing exactly 4 ones? exactly 6 ones?
4. If we toss a coin 10 times, what is the probability that all will be heads? that there will be exactly one tails? 2 tails? exactly 5 tails?
5. A drunkard is walking along a road from the pub to his house, which is located 1 mile north of the pub. Every step he makes can be either to the north, taking him closer to home, or to the south, back to the pub — and it is completely random: every step with can be north of south, with equal chances. What is the probability that after 10 steps, he will move
 - (a) 10 steps north
 - (b) 10 steps south
 - (c) 4 steps north
 - (d) will come back to the starting position
6. If you have a group of 4 people, and you need to choose one to go to a competition, how many ways are there to do it? if you need to choose 2? if you need to choose 3?
- *7. How many ways are there to select 5 students from a group of 12?
8. In a meeting of 25 people, each one shakes hands with each other. How many handshakes are there altogether?

9. If we draw 3 cards out of the deck, what are the chances that
- They will be all spades
 - They will be all aces
 - They will be ace of spades, queen of spades, and king of spades, in this order
 - They will be queen of spades, ace of spades, and king of spades, in this order
 - They will be ace, queen, and king of spades, in some order