# Math 7: Handout 9
# **Geometric Sequences continued.**

## ***Geometric Sequences - Infinite Sum***

The sum of a geometric sequence :

$$S = b_1 + b_2 + \ldots + b_n$$

$$S = b_1 \cdot \frac{q^n - 1}{q - 1}$$

For geometric sequences in which the common ratio -1 < q <1, we can consider the infinite sum.
For example: $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots = 2$
For an infinite sum, $S = \dfrac{b_1}{1 - q}$

## ***Homework***

1. Zeno's paradox: To walk across a room one has to go to half the distance $\frac{1}{2}$, then to one half of the ditance left or $\frac{1}{4}$, then $\frac{1}{8}$ and so on. This is an infinite number of steps compared to a finite amount of time available. But we know that anyone can get across the room in a timely manner. Hence the name: paradox. Can you explain the relationship between an infinite geometric sequence and Zeno's paradox?
2. What is the sum: $1 + x + x^2 + x^3 + x^4 + \ldots + x^{100}$
3. A geometric progression has 99 terms. the first term is 12 and the last term is 48. What is the 5-th term?
4. If we put one grain of wheat on the first square of the chessboard, two on the second, then four, eight, approximately how many grains of wheat will there be? (use $2^{10} = 1024$ or approximately $10^3$) Can you estimate the total volume of this wheat? Compare with the annual wheat harvest of the US which is about 2 billion bushels. (A grain of wheat is about $10mm^3$, a bushel is about 35 liters or $0.035m^3$)
5. Musicians use special notations for notes i.e. sound frequencies. Namely , they go as follows:
   $\ldots, A, A\#, B, C, C\#, D, D\#, E, F, F\#, G, G\#, A, A\#, \ldots$
   The interval between two notes in this list is called a halftone., the interval between A and the next A (or B and the next B, etc) is called an octave. Thus, one octave is 12 halftones. (If you have never seen it, read the description of how it works in Wikipedia) It turns out that the frequencies of the notes above form a geometric sequence: if the frequency of A in one octave is 440hz, then the frequency of $A\#$ is $440r$, frequency of B is $440r^2$ and so on.
   a. It is known that moving by one octave doubles the frequency. If the frequency of A in one octave is 440hz, then the frequency of A in the next octave is $2 \cdot 440 = 880hz$. Based on that, can you find the common ratio r of this geometric sequnce?
   b. Using the calculator, find the ratio of frequencies of A and E (such an interval is called a fifth). How close is it to $3 : 2$? Historic reference: the above convention for note frequencies is called equal temperament and is a relatively new tuning system. One of the early adopters was J.S.Bach who composed in 1722–1742 a collection of piano pieces called the Well-Tempered Clavier.
6. The sum of the first 20 terms of an arithmetic progression is 200, and the sum of the next 20 terms is -200. Find the sum of the first hundred terms of the progression.