MATH 7: HANDOUT 3
POWERS AND RADICALS

Today we discussed exponents and radicals and about converting between exponential form and radical form.

Exponents Laws

If a is a real number, n is a positive integer, we define $a^n = \underbrace{a \times a \times \cdots \times a}_{n\text{-times}}$

$$a^0 = 1$$
$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(ab)^n = a^n b^n$$
$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$
$$a^{-n} = \frac{1}{a^n}$$
$$(a^m)^n = a^{mn}$$

Why do we assume that $a^0 = 1$? We can see it from the following argument:

$$a^1 = a^{1+0} = a^1 a^0$$

Therefore, we must have the following equality (if we want to maintain the properties of exponents – and we do!):

$$a = a \cdot a^0$$
$$a^0 = 1$$

Now, why do we assume that $a^{-n} = \frac{1}{a^n}$? We can see it from the following argument:

$$a^{-n} \cdot a^n = a^{-n+n} = a^0 = 1.$$

Therefore, dividing both parts of the equation by a^n, we get:

$$a^{-n} = \frac{1}{a^n}.$$

Radicals

Now, what should the fractional powers be? Let us figure out what $a^{1/2}$ is. We will use similar logic as above:

$$a^{1/2} \cdot a^{1/2} = a^1 = a.$$

Therefore, $(a^{1/2})^2 = a$; from here, we can see that $a^{1/2} = \sqrt{a}$. Similarly, we can find that $a^{1/n} = \sqrt[n]{a}$. In general, we have the following properties:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m, n \neq 0$$
$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

Note that the exponential and radical laws work for multiplication and division. They do not work for addition and subtraction!

$$\sqrt[n]{a+b} \neq \sqrt[n]{a} + \sqrt[n]{b}$$
$$\sqrt[n]{a-b} \neq \sqrt[n]{a} - \sqrt[n]{b}, n \neq 0$$

HOMEWORK

1. The difference between two numbers is $\frac{5}{12}$. If $\frac{3}{4}$ of the larger number is $\frac{3}{8}$ more than $\frac{1}{2}$ of the smaller, find the larger number.

2. Simplify the following:

(a) $\frac{10^{-4}}{5^{-4}}$
(b) $\frac{3^2 \cdot 6^{-3}}{10^{-3} \cdot 5^2}$
(c) $(p^2q^3)(q^4p^6)$
(d) $\frac{(-x)^9}{x^{15}}$

3. Simplify the following expressions. When you have roots and radicals, reduce the expression to the product/ratio of radicals of simplest numbers. For example:

$$\sqrt{240} = \sqrt{16 \cdot 5 \cdot 3} = 4\sqrt{5}\sqrt{3}.$$

(a) $\sqrt{12}$
(b) $\sqrt{135}$
(c) $\sqrt{48}$
(d) $\sqrt[4]{243}$
(e) $\sqrt[3]{1250}$
(f) $\sqrt[3]{432}$

4. Calculate:

(a) $27^{2/3} \div 9^{3/2}$
(b) $16^{3/4} \cdot 125^{2/3}$
(c) $64^{3/4} \div 2^{3/2}$
(d) $81^{3/2} \cdot 27^{4/3} \div 3^{1/2}$

5. Simplify the following expressions with radicals. Assume all variables stand for positive numbers.

Example 1:

$$\sqrt[3]{x^4} = \sqrt[3]{x^3 \cdot x} = \sqrt[3]{x^3} \cdot \sqrt[3]{x} = x\sqrt[3]{x}.$$

(a) $\sqrt{72n^7}$
(b) $\sqrt{63u^3v^5}$
(c) $\sqrt{98a^7b^5}$
(d) $\sqrt[3]{24x^7}$
(e) $\sqrt[4]{64q^{10}}$
(f) $\sqrt[3]{x^4}$
(g) $\sqrt[4]{x^7}$
(h) $\sqrt{63u^3v^5}$
(i) $\sqrt{\frac{18p^5q^7}{32pq^2}}$

6. John takes 15 min to walk from school to the bus station. Jim takes 20 min to walk from the school to the bus station. If the difference in their speeds is 2 km/h, how far is the station from the school?

7. A square has diagonal length of 2. What is the square's side length?

8. Given an equilateral triangle with side 3, what is the altitude and what is the area of the triangle?