

Math 6: Homework 2.12

Odd and even

1) In the morning there were 5 space jets at the space port of the planet Pandora. During the day, several more jets landed, and a few jets departed from the port. The jets always arrived in groups of 2 or 4 and departed in groups of 2. In the evening, the air dispatcher counted 60 jets at the space port. Prove that the dispatcher miscalculated the number of jets.

2) While visiting the Knights and Knaves Island, Kevin the SchoolNova student met an islander. Kevin asked the islander how old he was. The islander replied, "If you multiply my age by 18, you get 1421." Kevin immediately recognized that the islander was a liar - why?

3) Savir the Junior Hacker reprogrammed the elevator in the 100-story Boogle Corporation building: only two buttons are currently working. The first button sends the elevator 8 floors up, and the second one 6 floors down. (The elevator will not move if it is asked to go above the 100th floor or below the 1st floor.) (a) The company's CEO is currently drinking coffee on the first floor. (There is no lobby floor in the building). Can he take the elevator to the 95th floor? If so, show how. If not, explain why. (b) Can he take the elevator to the 96th floor? If so, show how. If not, explain why

4) The numbers 1 through 10 are written on the blackboard in a row, with spaces left between them. Adam and Bill are playing the following game: on his turn, each player puts either + or – between two numbers. After all signs are written (so they get something like $1 + 23 + 4 + 56 \dots$), the total is computed. If it is even, Adam wins; if it odd, Bill. What is the best strategy for Adam? Should he take the first turn or leave the first turn to Bill?

5) The integers from 1 to 22 are written on the board in a row. Can you insert plus and minus signs between them in such a way as to get an expression that is equal to 0?

6) A train consists of a locomotive and five cars marked I, II, III, IV and V. In how many ways can you rearrange the cars, in such a way that car I is always closer to the locomotive than car II?

7) Can you connect 2021 computers with cables so that each computer is connected to exactly 3 other ones? [Hint: how many cables you would need?]