Math 6: Homework 2.9:

Factorization and Identities

When handling with large algebraic expressions, it is often possible to simplify them. One way of doing this is by **factorization**. As its name suggests, this method consists of finding a common factor in two or more terms. For example, in the following expression

$$7x + 9x - 5x$$

the three terms share the common factor x. Therefore, we can rewrite this expression as:

$$7x + 9x - 5x = (7 + 9 - 5)x = 11x.$$

In general, we will have the following identities:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

1. Simplify the following fractions and show the answer in the exponent form:

   a) $\left(\frac{3a^5b^2}{21ab}\right)^2 \cdot \frac{7^4}{a^{16}b^2}$ b) $\frac{3^5 \cdot 3^{-5}}{3^9}$ c) $\frac{1}{x-1} - \frac{2}{2x-1}$ d) $b - \frac{ab}{a-b}$

2. Factor the following expressions:

a) $(a + 4)^2 - 8(\mathrm{a} + 3) + 8$

b) $(b + 2)^2 - (\mathrm{b} + 4)(\mathrm{b} - 4)$

c) $(5c - 3)^2 + (12\mathrm{c} - 4)^2 - 4\mathrm{c}$

3. Show that the left-hand side (LHS) = right-hand side (RHS):

   a) $(a + b)^2 + c(a + b) = (a + b)(a + c) + (a + b)b$

   b) $x^2(x + 1) - x - 1 = x(x + 1)^2 - (x + 1)^2$

4. Find three consecutive integer numbers such that the sum of the first, twice the second, and three times the third is −70. (write and solve the equation)

5. Sixty more than nine times a number is the same as two less than seven times the number. Find the number.