MATH 6: HOMEWORK: PERMUTATIONS

An announcement: You can register for Math Kangaroo until December 15. All the info is on: https://schoolnova.org/nova/node/728

In general, if we are ordering k objects from a collection of n so that no repetitions are allowed, then this is referred to as a *permutation* of k objects from the collection of n, the number of ways to make such a selection of permutations is called ${}_nP_k$, and

$$ {}_{\boldsymbol{n}}\boldsymbol{P}_{\boldsymbol{k}} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{k})!} $$

In particular, if we take $k = n$, it means that we are selecting one by one all n objects — so this gives the number of possible ways to order n objects:

$$ {}_{\boldsymbol{n}}\boldsymbol{P}_{\boldsymbol{n}} = \boldsymbol{n}! = n(n-1)\ldots\cdot 2\cdot 1 $$

We read $n!$ as "n factorial". By convention, $0! = 1$, similar to the way that $x^0 = 1$.

For example: there are 52! ways to mix the cards in the usual card deck.

Note that the number $n!$ grow very fast: $2! = 2$, $3! = 6$, $4! = 2\cdot 3\cdot 4 = 24$, $5! = 120$, $6! = 620$

For example:

$$ {}_6P_4 = 6\cdot 5\cdot 4\cdot 3 = \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1} = \frac{6!}{2!} $$

Circular Permutations: The number of permutations of n elements in a circle with n places is **(n – 1)!**

That is since the circle can be rotated.

Additionally, if we can flip over the circle (we can do it with a beads as an example, not with a table). The number of permutations is:

$$ P'_n = \frac{1}{2}(n-1)! $$

1. A sly elementary school teacher decides to play favorites without telling anyone. If they have 15 students in their class, in how many ways can they choose a favorite student, a second favorite student, and a third favorite student?

2. Compute $\frac{6!}{3!}$, $6! - 3!$, ${}_5P_2$, ${}_5P_3$

3.

   a. How many ways are there to draw 3 cards from a 52-card deck? (Order matters: drawing first king of spades, then queen of hearts is different from drawing them in opposite order).

   b. How many ways are there to draw 3 cards from a 52-card deck if after each drawing we record the card we got, then return the card to the deck and reshuffle the deck? (As before, order matters.)

   c. We draw 3 cards from a 52-card deck, and after each drawing we record the card we got, then return the card to the deck and reshuffle the deck. What is the probability that all 3 drawn cards are different?

   a. sit in a different order every day for a year? How about two years?

4.

   a. How many 5s are there in the prime factorization of the number 100!? How many 2s?

   b. In how many zeroes does the number 100! end?

5. 10 people must form a circle for some dance. In how many ways can they do this?

6. In this problem, you have to express your answer as a simplified exponent (you do not have to compute numerically the expressions that you find). Simplify the expressions below using the power laws: $(a \times b)^n = a^n \times b^n$, $(a^n)^m = a^{n \times m}$, $a^n a^m = a^{n+m}$, $a^n / a^m = a^{n-m}$, $a^{-n} = 1/a^n$ and $a^0 = 1$.

   (a) $\frac{2^5 4^4}{2^7}$ (b) $6^5 \times 3^{-4}$ (c) $\frac{5^{-2}}{5^{-4}}$

7. Simplify: $\left(2^{10} \div (2^3)^3 + (2^2)^{2^2} + (3^8)^9 + ((3^2)^4)^9 - ((2^7)^3)^2\right) \div (1 + 2^6 + 4^3 + 3^{72} - 2^{41})$