MATH 6
HANDOUT 6: SETS PART 2

New material introduced today:

We say that set A is a *subset* of B (notation: $A \subseteq B$) if every element of A is also an element of B: $x \in A \Rightarrow x \in B$. Note that A can be equal to B. (Sometimes the notation $A \subset B$ is used, in which case it may or may not include equality, but we will use $\subseteq$ for both cases.)

Logically, we write the definition of $A \subseteq B$ as, for all x, $x \in A \implies x \in B$.

Additionally, it is useful to note that $(A \subseteq B)\,\text{AND}(B \subseteq A)$ means that $A = B$.

Counting

We denote by $|A|$ the number of elements in a set A (if this set is finite). For example, if $A = \{a, b, c, \dots, z\}$ is the set of all letters of English alphabet, then $|A| = 26$.

If we have two sets that do not intersect, then $|A \cup B| = |A| + |B|$: if there are 13 girls and 15 boys in the class, then the total is 28.

If the sets do intersect, the rule is more complicated:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

1. 150 people at a Van Halen concert were asked if they knew how to play piano, drums or guitar.
- 18 people could play none of these instruments.
- 10 people could play all three of these instruments.
- 77 people could play drums or guitar but could not play piano.
- 73 people could play guitar.
- 49 people could play at least two of these instruments.
- 13 people could play piano and guitar but could not play drums.
- 21 people could play piano and drums.

How many people can play piano? drums?

2. Find sets A,B,C if you know that: $A \cup B = \{1, 3, 4, 5, 7\}$ $B \cup C = \{1, 2, 4, 5, 6, 8, 9\}$

$(A \cup B) \cap C = \emptyset$ $(B \cup C) \cap A = \{1, 5\}$

3. Find A if you know that: $A \cup \{5, 7\} = \{3, 5, 7, 8\}$ $A \cap \{1, 2, 5, 7\} = \{5, 7\}$

4. For each of the sets below, draw it on the number line and find its complement (=opposite):

(a) [-5, 5] (b) $(-\infty, 2.5) \cap [1.2, +\infty)$ (c) 1

***5.** (a) Let $S = \{1, 2, 3, 4, 5\}$ and let $A \subseteq S$ such that A is a subset of exactly 4 subsets of S, including S itself and A itself. Can you determine how many elements are in A?

(b) Let $T_n = \{1, 2, 3, ..., n\}$, and similarly $T_k = \{1, 2, 3, ..., k\}$, with $k < n$. How many subsets of T_n is T_k a subset of? Don't forget to count T_n and T_k themselves.

***6.** Let A be the set of all ordered pairs of numbers (x, y) such that $0 < x < 1$, and B be the set of all ordered pairs of numbers (x, y) such that $0 < y < 1$.

(a) Is $(0.5, 1)$ in A? Is it in B?

(b) Determine whether the following points are in $A \cup B$: $(0.1, 0.2)$, $(0.1, 2.5)$, $(1.1, 1.1)$, $(-2, 0.5)$.

(c) Prove $(r, s) \in (A \cup B) \iff (1 - s, 1 - r) \in (A \cup B)$.