MATH 6: HOMEWORK 4
LOGIC VARIABLES AND TRUTH TABLES

On this class we will focus on learning about logic variables, truth tables and how to use these two to simplify complicated logic problems.

Logical variables: can only take two values, True (T) or False (F).
Basic logic operations:

NOT (for example, NOT *A*): true if *A* is false, and false if *A* is true.

AND (for example *A* AND *B*): true if both *A, B* are true, and false otherwise

OR (for example *A* OR *B*): true if at least one of *A, B* is true, and false otherwise. Some- times also called "inclusive or" to distinguish it from the "exclusive or" described below

IF: when someone says "if A then B", and A is false, do you think he lied? for example, is the statement below true?

"if sky is green, then 2+2=5"

As in usual algebra, logic operations can be combined, e.g. (*A* OR *B*) AND *C*.

Truth tables: If we have a logical formula involving variables *A, B, C, . . .* , we can make a table listing, for every possible combination of values of *A, B, . . .* , the value of our formula. The following are the truth tables for the operations NOT, AND, OR, IF:

AND is True if both statements are True
OR is True if at least one statement is True

A	NOT *A*
T	F
F	T

A	*B*	*A* AND *B*
T	T	T
T	F	F
F	T	F
F	F	F

A	*B*	*A* OR *B*
T	T	T
T	F	T
F	T	T
F	F	F

Truth tables provide the easiest way to prove complicated logical rules: if we want to prove that two formulas are equivalent (i.e., always give the same answer), make a truth table for each of them, and if the tables coincide, they are equivalent.

They are also useful in solving the problems about knights and knaves. Recall the problem from last time, with two inhabitants, Zed and Alice. Zed tells you, 'I am a knight **or** Alice is a knave.' Alice tells you, 'Of Zed and I, exactly one is a knight.' We could solve it by making the following table:

Zed	Alice	Zed: Z is a knight <u>or</u> A is a knave	Alice: Of Z and A, exactly one is a knight
Knight (T)	Knight (T)	T	F
Knight (T)	Knave (F)	T	T
Knave **(F)**	Knight **(T)**	**F**	**T**
Knave (F)	Knave (F)	T	F

After listing all the possible combinations at the left, we fill in the right part of the table comparing possible combinations with the statements that were made. After completing the table, we look for the raw in which the left and the right parts are the same. In the example above that is a third raw. So, Zed is a knave and Alice is a knight.

Homework

1. Many trucks carry the message: “If you do not see my mirrors, then I do not see you.” Can you rewrite it in an equivalent form without using the word “not”?

When doing the problems below, ***remember to use the truth tables to prove your answer!***

2. On the island of Knights and Knaves, you meet three inhabitants: Bozo, Carl, and Joe. Bozo says that Carl is a knave. Carl tells you, `Of Joe and I, exactly one is a knight.' Joe claims, `Bozo and I are different.' Can you determine who is a knight and who is a knave?

3. Prove using truth tables that NOT (A AND B) is the same as (NOT A) OR (NOT B).

4. King of the island has three boxes: one gold, one silver, and one lead. He tells you that there is a new cool sword in one of them, and that you must find in which one. There are statements at each box:

Gold	Silver	Lead
The sword is in this box	The sword is not in this box	The sword is in the Gold box

You know that at least one statement is true, and one is false. In which of them is a sword?

5. Define a new logical operation, XOR (exclusive or) as follows: A XOR B is true if exactly one of A; B is true, and false otherwise.

(a) Write the truth table for A XOR B.

(b) Can you express XOR using only AND; OR, and NOT?

***6.** (This starred problem is quite difficult; you can try just to read about it. It is called “The Hardest Logic Puzzle Ever”)

Three gods A, B, and C are called, in no particular order, True, False, and Random. True always speaks truly, False always speaks falsely, but whether Random speaks truly or falsely is a completely random matter. Your task is to determine the identities of A, B, and C by asking three yes-no questions; each question must be put to exactly one god. The gods understand English, but will answer all questions in their own language, in which the words for *yes* and *no* are *da* and *ja*, in some order. You do not know which word means which..