

## MATH 6: HOMEWORK 15

### ARITHMETIC SEQUENCE

A sequence of numbers is an arithmetic sequence if the difference between consecutive terms is the same number, the **common difference**, let's call it  $d$ .

For example, let's consider the sequence: 1, 5, 9, 13, 17, ...

The first term in the sequence is  $a_1 = 1$ , the second is  $a_2 = 5$ , and so on. The difference is  $d = 4$ .

What is the  $n^{\text{th}}$  term? For example what is  $a_{100}$ ?

$$a_1 = 1$$

$$a_2 = a_1 + d = 1 + 4 = 5$$

$$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d = (1 + 4) + 4 = 1 + 2 \times 4 = 9$$

$$a_4 = a_3 + d = (a_2 + d) + d = ((a_1 + d) + d) + d = a_1 + 3d = 1 + 3 \times 4 = 13$$

....

$$a_n = a_1 + (n - 1)d$$

$$\text{So } a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$$

### PROPERTY OF AN ARITHMETIC SEQUENCE

A property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

To prove this we can write:

$$a_n = a_{n-1} + d$$

$$a_n = a_{n+1} - d$$

Add them up and we have  $2a_n = a_{n-1} + a_{n+1}$ , divide by 2 and we get  $a_n = \frac{a_{n-1} + a_{n+1}}{2}$

To find the common difference between 2 terms  $a_s$  and  $a_t$

$$d = \frac{a_s - a_t}{s - t}.$$

### SUM OF AN ARITHMETIC SEQUENCE

$$S = a_1 + a_2 + a_3 + \dots + a_n = n \times \frac{a_1 + a_n}{2}$$

To prove this, we write the sum in 2 ways, in increasing and decreasing order:

$$S = a_1 + a_2 + a_3 + \dots + a_n$$

$$S = a_n + a_{n-1} + a_{n-2} + \dots + a_1$$

Adding up, we notice that  $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \dots$

$$2S = (a_1 + a_n) \times n \text{ and dividing by 2 gives us } S = n \times \frac{a_1 + a_n}{2}.$$

### HOMEWORK PROBLEMS

1. Write the first 5 terms of an arithmetic sequence if  $a_1 = 7$  and  $d = 2$ .
2. What are the first 2 terms for the sequence  $a_1, a_2, -9, -2, 5, \dots$
3.  $a_{10} = 131$  and  $d = 12$ . What is  $a_1$ ?
4.  $a_5 = 27$  and  $a_{27} = 60$ . Find the first term and the common difference.
5. Find the common difference in an arithmetic sequence if the  $9^{\text{th}}$  term is 18 and the  $11^{\text{th}}$  term is 44.
6. Find the sum of the first 100 terms if  $a_1 = 10$  and  $a_{100} = 150$ .
7. Find the sum of all odd numbers from 1 to 2011.
- \*8. Can you continue the following sequence:  $-2, 1, 6, 13, 22, \dots$  [Hint: look at the differences of successive terms!]
9. Find the shortest distance from the origin  $(0, 0)$  to the line given by the equation  $y = -2x + 8$ .
10. Compute the area of the rectangle  $ABCD$  if  $A$  is at  $(0, 0)$ ,  $B$  at  $(2, 3)$ , and  $D$  at  $(-6, 8)$ . [It can be done in more than one way.]