MATH 6
ASSIGNMENT 9: LET'S PLAY CASINO!
DECEMBER 3, 2023

BASIC PROBABILITY

Basic probability rule:

$$P(win) = \frac{\text{number of winning outcomes}}{\text{total number of possible outcomes}}$$

(assuming all outcomes are equally likely!)

For example, probability of drawing a spade card out of the standard deck is

$$P = \frac{13}{52} = \frac{1}{4}$$

COMPLEMENT RULE

If probaility of some event is P then the probability that this event will **not** happen is $1 - P$.

For example, if we draw a card from the deck then the probability that it is **not** a spade is $1 - \frac{1}{4} = \frac{3}{4}$.

ADDITION RULE

Suppose we are drawing a card from the deck of 52 cards and ask: what is the probability of getting either queen or king. Since there are 4 queens and 4 kings, which makes it 8 cards total, we can write

$$P(\text{queen or king}) = \frac{(4+4)}{52} = \frac{4}{52} + \frac{4}{52} = P(queen) + P(king)$$

In general, we have the following rule:

$$\text{P(A or B) = P(A) + P(B)}$$

if A and B can't happen together.

PRODUCT RULE

If we do two trials (e.g., rolling a die twice), then the probability of getting result A in the first trial and result B is the second one is

$$P(\text{A, then B}) = P(A)P(B)$$

if results of the second trial **do not depend** on the first one.

EXAMPLE: TOSSING A COIN

Question. If toss a coin 10 times, what is the probability that all will be heads?

Answer. $\left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}}$ (using calculator, one can compute that it is $1/1024 \approx 0.001$, or $1/10$ of 1%).

Question. If toss a coin 10 times, what is the probability that all will be tails?

Answer. The same.

Question. If we toss a coin 10 times, what is the probability that **at least one** will be heads?

Answer. Unfortunately, there are very many combinations which give at least one heads. In fact, it is easier to say which combinations **do not** give at least one heads: there is exactly one such combination, all tails; probability of getting this combination is, as we computed, $1/2^{10} = \frac{1}{1024}$. The remaining combinations will give at least one heads; thus probability of getting at least one heads is $1 - \frac{1}{1024} = \frac{1023}{1024} \approx 0.999$.

1. We take the standard card deck and draw one card. What is the probability that the card will be
   (a) Queen of hearts
   (b) Either a queen or a hearts card
   (c) A red card
   (d) A picture card (a jack, queen, king, ace)
   (e) A picture card other than the queen of hearts

2. (a) What is the probability that when we toss a coin 4 times, there will be no heads?
   (b) A and B are playing the following game. They toss a coin 4 times; if there are no heads, A wins, and B pays him $10. Otherwise A loses and he pays $1 to B.
   Would you prefer to play for A or for B in this game?

3. (a) What is the probability that when we roll two dice, at least one will be a 6?
   (b) A and B are playing the following game. They roll two dice; if at least one is a 6, A wins, and B pays him $5. Otherwise A loses and he pays $1 to B.
   Would you prefer to play for A or for B in this game?

4. (a) What is the probability that if we roll 3 dice, all the numbers will be odd?
   (b) A and B are playing the following game. They roll 3 dice; if all numbers are odd, A wins, and B pays him $5. Otherwise A loses and he pays $1 to B.
   Would you prefer to play for A or for B in this game?

5. Supposing that there are equal chances of a boy or a girl being born, what is the probability that the first five babies born next Saturday morning at the St. Charles Hospital will be girls? That at least one of them five will be a girl?

6. In a certain club of 30 people, they are selecting a president, vice-president, and a treasurer (they all must be different people: no one is allowed to take two posts at once). How many ways are there to do this?

7. In a group of 100 students, 28 speak Spanish, 30 speak German, 42 speak French; 8 students speak Spanish and German, 10 speak Spanish and French, 5 speak German and French and 3 students speak all 3 languages. How many students do not speak any one of the three languages?
   [Note: when it says that 28 students speak Spanish, this includes the 8 who speak Spanish and German; similarly for all other combinations.]

8. How many whole numbers between 1–1000 are divisible by 3? by 5? by 15? are not divisible by either 3 or 5?

9. (a) What is the probability that if we roll 2 dice, the sum will be at most 7?
   (b) A and B are playing the following game. They roll 2 dice; if the sum is at most 7, A wins, and B pays him $1. Otherwise A loses and he pays to B $1.
   Would you prefer to play for A or for B in this game?

10. (a) What is the probability that if we roll 3 dice, all the numbers will be different?
    (b) A and B are playing the following game. They roll 3 dice; if all numbers are different, A wins, and B pays him $2. Otherwise A loses and he pays to B $3.
    Would you prefer to play for A or for B in this game?

11. (a) In a class of 25 students, everyone chooses a date (e.g., March 13). How many combinations are possible? (Students only choose month and day, not year; February 29th is not allowed, so there are 365 different possibilities. Also, it matters who had chosen which day: combination where Bill has chosen March 12 and John, June 15 is considered different from the one where Bill has chosen June 15 and John March 12.)
    (b) In the same situation, how many such combinations are possible if we additionally require that all dates must be different?
    *(c) Suppose now that each of these 25 students has chosen a date at random, not knowing the choices of others. What is the probability that all of these dates will be different? That at least 2 will coincide?