

## MATH 6. ASSIGNMENT 7: SETS

NOVEMBER 5, 2023

### SETS

By word *set*, we mean any collection of objects: numbers, letters,... Most of the sets we will consider will consist either of numbers or points in the plane. Objects of the set are usually referred to as *elements* of this set.

Sets are usually described in one of two ways:

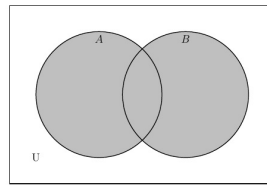
- By explicitly listing all elements of the set. In this case, curly brackets are used, e.g.  $\{1, 2, 3\}$ .
- By giving some conditions, e.g. “set of all numbers satisfying equation  $x^2 > 2$ ”. In this case, the following notation is used:  $\{x \mid \dots\}$ , where dots stand for some condition (equation, inequality, ...) involving  $x$ , denotes the set of all  $x$  satisfying this condition. For example,  $\{x \mid x^2 > 2\}$  means “set of all  $x$  such that  $x^2 > 2$ ”.

Other notation:

$x \in A$  means “ $x$  is in  $A$ ”, or “ $x$  is an element of  $A$ ”

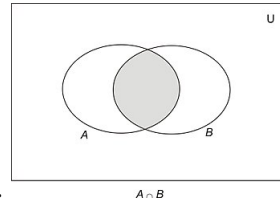
$x \notin A$  means “ $x$  is not in  $A$ ”

$A \cup B$ : union of  $A$  and  $B$ . It consists of all elements which are in either  $A$  or  $B$  (or both):



$$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}.$$

$A \cap B$ : intersection of  $A$  and  $B$ . It consists of all elements which are in both  $A$  and  $B$ :



$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}.$$

$\bar{A}$ : complement of  $A$ , i.e. the set of all elements which are not in  $A$ :  $\bar{A} = \{x \mid x \notin A\}$ .

## HOMEWORK

1. Consider the operation **NOR** which is just the opposite of **OR**: it returns **1** or **T** only if both **A** and **B** are **0** or **F**. Using only the component **NOR**, see if you can create circuits equivalent to **AND**, **OR**, and **NOT** similar to what we did with **nand**.
2. Using only **AND**, **NOT**, and **OR**, produce a three-input **AND** circuit, i.e., the output is **F** unless all three inputs are **1** or **T**. (You do not have to use all three circuit elements.)
3. Using only **AND**, **NOT**, and **OR**, produce a three-input **OR** circuit, i.e., the output is **1** or **T** if any of the inputs is **1** or **T**.
4. If **Al** comes to a party, **Betsy** will not come. **Al** never comes to a party where **Charley** comes. And either **Betsy** or **Charley** (or both) will certainly come to the party.  
Based on all of this, can you explain why it is impossible that **Al** comes to the party?

5. Let

$A$ =set of all people who know French

$B$ =set of all people who know German

$C$ =set of all people who know Russian

Describe in words the following sets:

(a)  $A \cap B$     (b)  $A \cup (B \cap C)$     (c)  $(A \cap B) \cup (A \cap C)$     (d)  $C \cap \bar{A}$ .

6. Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.

Denote:

$H$ =set of all hearts cards

$Q$ =set of all queens

$R$ =set of all red cards

Describe by formulas (such as  $H \cap Q$ ) the following sets:

all red queens

all black cards

all cards that are either hearts or a queen

all cards other than red queens

How many cards are there in each set?

7. In a class of 25 students, 10 students know French, 5 students know Russian, and 12 know neither. How many students know both Russian and French?
8. Simplify the following expressions.

(a)  $\frac{6^6 \cdot 2^2}{3^9 \cdot 2^{17}} =$

(b)  $\frac{(15^3)^3 \cdot 3^9}{5^{15}} =$

(c)  $(7^7 \cdot 7^3)^2 =$

(d)  $\frac{2^{-2}}{2^{-12}} =$