

MATH 6
ASSIGNMENT 3: KNIGHTS AND KNAVES

OCTOBER, 1 2023

PROBLEMS

Many of the questions of this assignment refer to the famous (among logic puzzle fans) island of Knights and Knaves. On this island, there are two kinds of people: Knights, who always tell the truth, and Knaves, who always lie. Unfortunately, there is no easy way of knowing whether a person you meet is a knight or a knave...

Copyright notice: most of these problems come from books of Raymond Smullyan. If you liked them, get his books in the library and you will find there many more puzzles of the same sort. You can also find a number of such puzzles online at <http://philosophy.hku.hk/think/logic/puzzles.php>

1. Find the greatest common divisor and least common multiple of 132 and 90.
2. Solve the following equation:

$$\frac{x-5}{x-2} = 9$$

3. (This question should be answered without using a calculator.) The maximal distance from Sun to Pluto is 7,375,927,931 km. Speed of light is about 300,000 km/sec. How long does it take for Sun's light to reach Pluto? (You do not need to give a precise answer — an approximate one like "about 2 minutes" would be fine.)
4. You meet two inhabitants: Bob and Betty. Bob claims that Betty is a knave. Betty tells you, "I am a knight or Bob is a knight." Can you determine who is a knight and who is a knave?
5. On the island of knights and knaves, you meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says, "Neither Zoey nor I are knaves." So who is a knight and who is a knave?
6. On the island of knights and knaves, you meet two inhabitants: Sue and Zippy. Sue says that Zippy is a knave. Zippy says, "I and Sue are knights." So who is a knight and who is a knave?
7. On the island of knights and knaves, you meet two inhabitants: Bart and Ted. Bart claims, "I and Ted are both knights or both knaves." Ted tells you, "Bart would tell you that I am a knave." So who is a knight and who is a knave?
8. A traveler to the island of Knights and Knaves meets a group of five people (call them A, B, C, D, E).
A says: "exactly one of us is a Knight"
B says: "exactly two of us are Knights"
C says: "exactly three of us are Knights"
D says: "exactly four of us are Knights"
E says: "all five of us are Knights"
Can you find out which of them are Knights?
9. You are in a maze on the island of knights and knaves. There are two doors: you know that one leads to freedom and one leads to certain doom. There are two guards nearby, and you happen to know that one is a knight and one is a knave, but you don't know who is who. They allow you to ask one of them a single question before you choose a door — what do you ask?