

**MATH 6: HANDOUT 16**  
**GEOMETRIC SEQUENCES**

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A sequence of numbers is a **geometric sequence** or **geometric progression** if the next number in the sequence is the current number times a fixed constant called the **common ratio** or  $q$ .

**Example:** The sequence 6, 12, 24, 48, ... is a geometric sequence because the next number is obtained from the previous by multiplication by  $q = 2$ .

We can also find the  $n$ -th term if we know the 1st term and  $q$ .

**Example:** What is  $a_{10}$  in the example above?

$$a_1 = 6$$

$$a_2 = a_1q = 6 \cdot 2 = 12$$

$$a_3 = a_2q = (a_1q)q = a_1q^2 = 6 \cdot 2^2 = 24$$

The pattern is:

$$a_n = a_1q^{n-1}$$

$$a_{10} = a_1q^9 = 6 \cdot 2^9 = 6 \cdot 512 = 3072$$

**Properties of a Geometric Sequence.** Any term is the **geometric mean** of its neighbors:

$$a_n = \sqrt{a_{n-1} \cdot a_{n+1}}$$

**Proof:**

$$a_n = a_{n-1}q$$

$$a_n = a_{n+1}/q$$

Multiplying these two equalities gives us:

$$a_n^2 = a_{n-1} \cdot a_{n+1}$$

from where we can get what we need.

PROBLEMS

- Write out the first four terms of each of the following geometric sequences, given the first term  $b_1$  and common ratio  $q$ .
  - $b_1 = 1$  and  $q = 3$
  - $b_1 = 1$  and  $q = \frac{1}{2}$
  - $b_1 = -10$  and  $q = \frac{1}{2}$
  - $b_1 = 27$  and  $q = -\frac{1}{3}$
- What are the first two terms of the geometric progressions  $a_1, a_2, 24, 36, 54, \dots$ ?
- Find the common ratio of the geometric progressions  $1/2, -1/2, 1/2, \dots$ . What is  $a_{10}$ ?
- A geometric progression has 99 terms, the first term is 12 and the last term is 48. What is the 50th term?