

MATH 6: HANDOUT 11
FACTORIALS AND PERMUTATIONS.

PERMUTATIONS

If we are choosing k objects from a collection of n so that a)order matters and b)no repetitions allowed, then there are

$$n(n - 1) \dots (k \text{ factors})$$

ways to do it.

In particular, if we take $k = n$, it means that we are selecting one by one all n objects — so this gives the number of possible ways to order n objects:

$$n! = n(n - 1) \dots 2 \cdot 1$$

(reads n factorial).

For example: there are $52!$ ways to mix the cards in the usual card deck.

Note that the number $n!$ grow very fast: $2! = 2$, $3! = 6$, $4! = 2 \cdot 3 \cdot 4 = 24$, $5! = 120$, $6! = 620$

MONTY HALL PROBLEM

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?"

Is it to your advantage to switch your choice?

Answer: yes!

Let us see what happens if you have chosen door No. 1. Then there are the following possibilities:

Car is behind door..	Probability	It is better to
1	1/3	stay
2	1/3	switch
3	1/3	switch

Thus, switching wins with probability $2/3$.

HOMework

In all the problems that ask you to compute something, it suffices to write an expression for the answer, e.g., $1/2^{11}$; it is not necessary to actually perform the multiplication.

1. About $1/6$ of Americans have blue eyes. If we choose 10 people at random, what is the probability that all of them have blue eyes? that none has blue eyes? that at least one has blue eyes?
2. A group of 6 club members always dine at the same table in the club; there are exactly 6 chairs at the table. They decided that each day, they want to seat in a different order. Can they keep this for a year? Two years?
3. How many ways are there to seat 15 students in a classroom which has 15 chairs? If the room has 25 chairs?
4. A puzzle consists of 9 small square pieces which must be put together to form a 3×3 square so that the pattern matches (this kind of puzzles is actually quite hard to solve!). It is known that there is only one correct solution. If you started trying all possible combinations at random, doing one new combination a second, how long will it take you to try them all?
5. 10 people must form a circle for some dance. In how many ways can they do this?
6. At a fair, they offer you to play the following game: you are tossing small balls in a large crate full of empty bottles; if at least one of the balls lands inside a bottle, you win a stuffed toy (worth about \$5). Unfortunately, it is really impossible to aim, so the game is just a matter of luck (or probability theory): every ball you toss has a 20% probability of landing inside the bottle.
 - (a) If you are given three balls, what is the probability that all three will be hits? That all three will be misses? That at least one will be a hit?
 - (b) Same questions for five balls.
 - * (c) They charge you 2 dollars for 3 balls, or 3 dollars for 5 balls. Which is a better deal? [Considering only from the point of view of the chances of winning, not the fun you are getting]