MATH 6
HANDOUT 5: SETS

SETS

Describing Sets. By word *set*, we mean any collection of objects: numbers, letters, ... Most of the sets we will consider will consist either of numbers or points in the plane. Objects of the set are usually referred to as *elements* of this set.

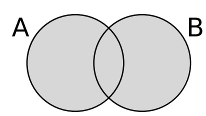
Sets are usually described in one of two ways:

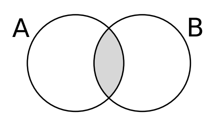
- By explicitly listing all elements of the set. In this case, curly brackets are used, e.g. $\{1, 2, 3\}$.
- By giving some conditions, e.g. "set of all numbers satisfying equation $x^2 > 2$". In this case, the following notation is used: $\{x \mid \dots\}$, where dots stand for some condition (equation, inequality, ...) involving x, denotes the set of all x satisfying this condition. For example, $\{x \mid x^2 > 2\}$ means "set of all x such that $x^2 > 2$".

Members of sets. Sometimes we might have to say whether the element belongs to the set or not. In this case the following notation is used:

- $x \in A$ means "x is in A", or "x is an element of A"
- $x \notin A$ means "x is not in A"

Set Operations. There are several operations that can be used to get new sets out of the old ones:

- $A \cup B$: *union* of A and B. It consists of all elements which are in either A or B (or both):
$$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}.$$


- $A \cap B$: *intersection* of A and B. It consists of all elements which are in both A and B:
$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}.$$


- $\overline{A}$: *complement* of A, i.e. the set of all elements which are not in A: $\overline{A} = \{x \mid x \notin A\}$.

Homework

1. Another one of Lewis Carroll's puzzles:

(a) My saucepans are the only things I have that are made of tin.
(b) I find all your presents very useful.
(c) None of my saucepans are of the slightest use.

2. And another one:

(a) No birds, except ostriches, are 9 feet high.
(b) There are no birds in this aviary that belong to anyone but me.
(c) No ostrich lives on mince pies.
(d) I have no birds less than 9 feet high.

3. Write the truth table for the formula: $\not P \text{ AND } (Q \text{ OR} (\text{NOT } R))$

4. If Al comes to a party, Betsy will not come. Al never comes to a party where Charley comes. And either Betsy or Charley (or both) will certainly come to the party.

Based on all of this, can you explain why it is impossible that Al comes to the party?

5. Let

- A=set of all people who know French
- B=set of all people who know German
- C=set of all people who know Russian

Describe in words the following sets:

(a) $A \cap B$ (b)$A \cup (B \cap C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) $C \cap \overline{A}$.

6. Let $A = \{1, 3, 5\}, B = \{3, 5, 6\}$, and $C = \{1, 3, 7\}$. Calculate the sets in the right hand side and in the left hand sides of the following expressions, and show that the results match:

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

7. Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.

Denote:

- H=set of all hearts cards
- Q=set of all queens
- R=set of all red cards

Describe by formulas (such as $H \cap Q$) the following sets:

- all red queens
- all black cards
- all cards that are either hearts or a queen
- all cards other than red queens

How many cards are there in each set?

8. In a class of 25 students, 10 students know French, 5 students know Russian, and 12 know neither. How many students know both Russian and French?