MATH 5: HANDOUT 13
NEGATIVE POWERS. SCIENTIFIC NOTATION.

Recall that for a positive integer n, we have defined

$$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ times}}$$

then

$$a^m a^n = a^{m+n}, \qquad a^m \div a^n = a^{m-n}$$

It turns out that there is only one way to define a^n for $n = 0$ and negative n so that these rules still work, namely:

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

For example, $3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

SCIENTIFIC NOTATION

Scientific notation is a convenient way to write very large numbers: instead of writing $2,000,000,000$ one can say "2 and then 9 zeros". Since writing a zero at the end is the same as multiplication by 10, we can also write the same number as

$$2 \times 10 \times \cdots \times 10 \qquad \text{(9 times)}$$

or, for short 2×10^9. Thus, we can write

$$2,000,000,000 = 2 \times 10^9$$

which is much shorter.

Similarly, we can write

$$\begin{aligned} 2,310,000,000 &= 231 \times 10 \times \cdots \times 10 \qquad \text{(7 times)} \\ &= 2.31 \times 10 \times \cdots \times 10 \qquad \text{(9 times)} \\ &= 2.31 \times 10^9 \end{aligned}$$

Such a form (a decimal with one digit before decimal point times 10 to some power) is called the *scientific notation*.

To write a number larger than 10 in scientific notation, you should:

1. Count how many digits the whole part has. The power of 10 will be number of digits minus 1.

2. Write down the digits of the number, but now put the decimal point after the first digit.

Example:

$$3412000 = 3.412000 \times 10^6 = 3.412 \times 10^6$$

SOLUTION FOR PROBLEM 6 FROM PREVIOUS HW

Problem 6: (from *101 puzzles in thought and logic*, by C. R. Wylie)

Clark, Jones, Morgan, and Smith are four men whose occupation are butcher, druggist, grocer, and policeman, though not necessarily in that order.

1) Clark and Jones are neighbors and take turns driving each other to work

2) Jones makes more money than Morgan

3) Clark beats Smith regularly at bowling

4) The butcher always walks to work

5) The policeman doesn't live near the druggist

6) The only time the grocer and the policeman ever meet is when the policeman arrested the grocer for speeding

7) The policeman makes more money than the druggist or the grocer

What is each man's occupation?

SOLUTION

- From 1) and 4) The butcher always walks to work, so the butcher is not Clark or Jones (who take turns driving each other).
- From 5) and 6) neighbors (Clark and Jones) are not policemen as policeman only met the grocer once and does not live close to the druggist. Consequently, the neighbors are identified as the grocer and the druggist.
- From 2) and 7) As Jones (druggist or grocer) makes more money than Morgan (policeman or butcher), and the policeman makes more money than the druggist and the grocer, Morgan is not a policeman. So, he must be butcher and Smith must be policeman.
- From 3) and 6) Clark is not grocer as policeman (Smith) regularly plays with Clack, but met the grocer only once. So, Clark is the druggist. And Jones is the grocer.

So,

Smith - policeman
Morgan - butcher
Jones - grocer,
Clark - druggist.

HOMEWORK

1. If $a = 2^{-13}3^9$, $b = 2^{11}3^{-7}$, what is the value of
   (a) ab
   (b) $\dfrac{a}{b}$

2. In how many zeroes does the number $4^{15}5^{26}$ end?

3. Simplify:
   (a) $4c^2 \cdot c^3$
   (b) $\dfrac{8dg^2}{3d^3g^4}$
   (c) $\dfrac{26a^2b}{65a^3b^2c^3}$
   (d) $\dfrac{9a^7b^6}{45a^3b}$
   (e) $\dfrac{3^4a^{20}b^8}{(21ab)^4} \cdot \dfrac{7^4}{a^{16}b^2}$

4. Let $x = a^3 \cdot b^2$, $y = \dfrac{b^5}{a^2c^4}$, and $z = \dfrac{c^3}{ab}$. Express in terms of a, b, c:
   (a) xyz
   (b) $\dfrac{x}{y}$

5. Suppose \$100 is deposited into an account and the amount doubles every 8 years. How much will be in the account after 40 years? Express your answer using powers.

6. At the beginning of an epidemic, 50 people are sick. If the number of sick people triples every other day, how many people will be sick at the end of 2 weeks? Express your answer using powers.

7. Write the following numbers using scientific notation.
(a) the distance from Earth to Pluto is $\approx$ 7,527,000,000 km;
(b) the distance from Earth to the star Sirius is $\approx$ 81,900,000,000,000 km;
(c) the distance from Earth to Vega is $\approx$ 249,500,000,000,000 km;
(d) the distance from Earth to the Andromeda Nebula is $\approx$ 2,000,000,000,000,000,000,000 km.
(e) the area of the Pacific Ocean is $\approx$ 178,684,000,000 km^2

8. ***(Science)** Scientific notation is very useful for very small numbers. For example, weight of one atom of hydrogen is about 1.66×10^{-24} gram — or

$$0.00000000000000000000000166\ gr$$

About how many hydrogen atoms are there in one gram of hydrogen?