Classwork 7.

Direct and invers proportionality.

Last week we discuss proportions, which involve two equal ratios. But what if there are many equal ratios? For example, if I walk at a constant speed, how does the time of my walk correlate with the distance I've covered?

Fill the table:

My speed is 3 km per hour

t	0.5	1	1.5	2	2.5	3	3.5	4
S								

My speed is 5 km per hour

t	0.5	1	1.5	2	2.5	3	3.5	4
S								

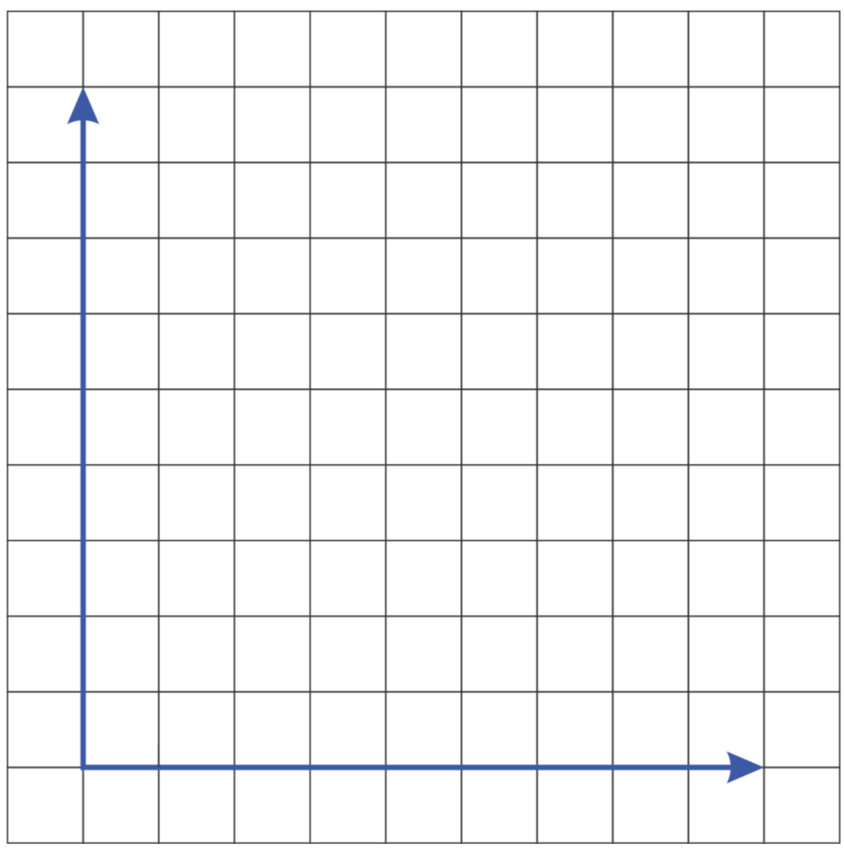
$$\frac{S}{t} = v$$

As you can see, if the speed is constant, the ratio of a distance to time is always be the speed.

In other words, $S = 5\left({}^{km}/_{h}\right) * t(hour)$, 5 is a constant, and it's called a constant of proportionality. Longer travel $\Rightarrow$ further from the initial point, and the ratio between these two variables will be always the same, *distance*:*time* is speed. If the time three time longer, the distance will be three time greater as well. (When the speed is constant of cause). Two variables, *distance* and *time*, are dependent from each other, they are in the relation of proportionality.

We can write the relationship between the distance, time, and speed as

$$S = v \cdot t$$



Two variables (y and x) are related proportionally if

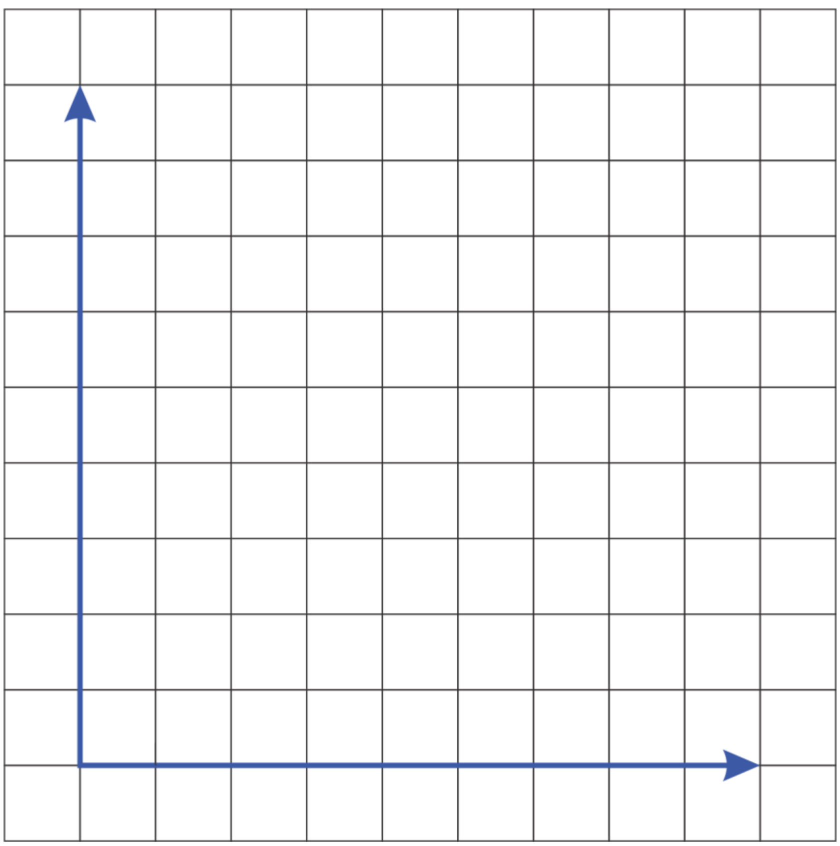
$$y = kx; \quad \frac{y}{x} = k, x \neq 0$$

We say that y is directly proportional to x, and coefficient of proportionality is k. Another example of direct proportionality is the circumference and the diameter of a circle, π is coefficient.

A notebook costs 3 dollars. How much I need to pay if I buy 3 notebooks, 5, 12? What variables are used, what is the relationship between them? Can a constant of proportionality be found?

The distance between my home and my work is 20 miles. Let's see, how the time and speed are corelated, if I ride a bike to work:

v	80	40	20	10	5	2	1	0.5	0.1
t									



This kind of correlation is called invers proportionality.

Exercises:

1. A squirrel is doing a stock of acorns for winter. Every 20 minutes it brings 2 acorns. How many acorns it will have in 40 minutes? 80 minutes?

Time (t)	20 minutes	40 minutes	80 minutes	2 hours
Number of acorns				

2. Bacteria are dividing every 30 minutes. I want to make yogurt and I put 1 bacterium in a cup of milk. How many bacteria will be there in the milk in 1 hour? in 2 hours? in 3 hours?

Time (t)	0.5 hour	1 hour	2 hours	4 hours
Number of bacteria				

Problems with proportions:

Problem 1. To prepare 6 large pizzas, the cook needs 2.5 kg of flour. How much flour does the cook need to prepare 8 pizzas? We can write the problem as follows:

6 pizzas → 2.5 kg

8 pizzas → x kg

We can create several proportions:

1. How many kilograms of flour are needed to make one pizza:

$$\frac{2.5\ kg.}{6} = \frac{x\ kg.}{8}$$

2. Flour consumption is proportional to the number of pizzas made, so if twice as many pizzas are made, twice as much flour should be used.

$$\frac{6}{8} = \frac{2.5\ kg.}{x\ kg.}$$

3. How many pizzas can be made with 1 kg of flour?

$$\frac{6}{2.5\ kg} = \frac{8}{x\ kg.}$$

For the first proportion:

$$\frac{2.5\ kg.}{6} = \frac{x\ kg.}{8}; \quad 8 \cdot 2.5 kg = 6 \cdot x\ kg.; \quad x = \frac{8 \cdot 2.5\ kg.}{6} = \frac{4 \cdot 2.5\ kg}{3} = \frac{10}{3} = 3\frac{1}{3} kg.$$

For the second and third:

$$\frac{6}{8} = \frac{2.5\ kg.}{x\ kg.};\quad 6 \cdot x\ kg = 8 \cdot 2.5kg;\quad x\ = \frac{8 \cdot 2.5\ kg.}{6} = \frac{4 \cdot 2.5\ kg}{3} = \frac{10}{3}\ = 3\frac{1}{3}kg$$

$$\frac{6}{2.5\ kg} = \frac{8}{x\ kg.};\ 6 \cdot x\ kg = 8 \cdot 2.5kg;\quad x\ = \frac{8 \cdot 2.5\ kg.}{6} = \frac{4 \cdot 2.5\ kg}{3} = \frac{10}{3}\ = 3\frac{1}{3}kg$$

6 pizzas → 2.5 kg
8 pizzas → x kg

Problem 2. 6 typists working 5 hours a day can type the manuscript of a book in 16 days. How many days will 4 typists take to do the same job, each working 6 hours a day?

$$6\ typists \cdot 5\ hours \rightarrow 16\ days$$
$$4\ typists \cdot 6\ hours \rightarrow x\ days$$

When writing a proportion, we must be careful to choose the right one: more typists, more hours a day, less time to get the job done.

$$\frac{6 \cdot 5}{4 \cdot 6} \neq \frac{16}{x};\quad \frac{6 \cdot 5}{4 \cdot 6} = \frac{x}{16};$$

$$\frac{5}{4} = \frac{x}{16};\qquad 4x = 16 \cdot 5;\qquad x = \frac{16 \cdot 5}{4} = 20\ days.$$

This problem can be solved without writing the proportion. Number of hours of typing for one typist needed to do the job is $16\ days \cdot 6\ typists \cdot 5\ hour\ per\ day$ should be equal to $x\ days \cdot 4\ typists \cdot 6\ hour\ per\ day$

$$16 \cdot 6 \cdot 5 = x \cdot 4 \cdot 6;\quad x = \frac{16 \cdot 6 \cdot 5}{4 \cdot 6} = 20\ days$$

Exercises:

1. The relationship between two variables is given in the table below. Is this relationship proportional? If so, what is the constant of proportionality?

a.

x	9	15	33	45	66
y	3	5	11	15	22

b.

x	3	2	5	4	6
y	9	4	25	16	36

c.

x	3	2	1	$\frac{1}{3}$	30
y	1	$\frac{3}{2}$	3	9	0.1

2. Are the following variables proportional?
   a. Speed and time of movement on a distance of 50 km.
   b. Speed and corresponding distance after 2 hours of driving.
   c. Price of the 1 notebook and the number of notebooks which can be bought with 24 dollars.
   d. Length and the width of the rectangle with the area of 60 cm^2.

3. A car travels 60 km during a certain time. How this time will change, if the speed will be increased 3 times?

4. The sorcerer used seaweed and mushrooms in a ratio of 5 to 2 when brewing a potion. How much seaweed does he need if there are only 450 grams of mushrooms?

5. A car travels from one city to another in 13 hours at a speed of 75 km/h. How long will it take if the car moves at a speed of 52 km/h?

6. Which of the following formulas describe the direct proportionality, inverse proportionality or neither of the two?

$$P = 5.2b; \quad K = \frac{n}{2}; \quad a = \frac{8}{b}; \quad M = m:5; \quad G = \frac{1}{4k};$$
$$a = 8q + 1; \quad c = 4:d, \quad 300 = v \cdot t; \quad ab = 18; \quad S = a^2$$

7. Peter’s time of the driving to work usually is 1 hour and 20 minutes. Yesterday was a bad weather and Peter reduced his speed by 10 km/h and reached his work in 1.5 hours. What is the distance between Peter’s house and his work?