Math 4b.

Exponent

Exponentiation is a mathematical operation, written as a^n, involving two numbers, the **base** *a* and the **exponent** *n*. When *n* is a positive integer, exponentiation corresponds to repeated multiplication of the base: that is, a^n is the product of multiplying *n* bases:

$$a^n = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n\ times}$$

In that case, a^n is called the ***n*-th power of *a***, or ***a* raised to the power *n***.
The exponent indicates how many copies of the base are multiplied together. For example, $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$. The base 3 appears 5 times in the repeated multiplication, because the exponent is 5. Here, 3 is the *base*, 5 is the *exponent*, and 243 is the *power* or, more specifically, *the fifth power of 3*, *3 raised to the fifth power*, or *3 to the power of 5*.

Properties of exponent:

1. If the same base raised to the different power and then multiplied:

$4^3 \cdot 4^5 = (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^8 = 4^{3+5}$

Or in a more general way:

$$a^n \cdot a^m = \underbrace{a \cdot a \dots \cdot a}_{n\ times} \cdot \underbrace{a \cdot a \dots \cdot a}_{m\ times} = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n+m\ times} = a^{n+m}$$

2. If the base raised to the power of *n* then raised again to the power of *m*:

$$(4^3)^5 = (4^3) \cdot (4^3) \cdot (4^3) \cdot (4^3) \cdot (4^3)$$
$$= (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4)$$

Or in a more general way:

$$(a^n)^m = \underbrace{a^n \cdot a^n \cdot \dots \cdot a^n}_{m\ times} = \underbrace{\underbrace{a \cdot a \cdot \dots \cdot a}_{n\ times} \cdot \dots \cdot \underbrace{a \cdot a \cdot \dots \cdot a}_{n\ times}}_{m\ times} = a^{n \cdot m}$$

Homework

1. Continue the sequence:

   $a.\ 1, 4, 9, 16 \ldots$ $b. 1, 8, 27, \ldots$ $c.\ 1, 4, 8, 16 \ldots$

2. Write the following numbers as a second power:

   Example: $25 = 5^2$

   25, 121, 144, 225

3. Find x so that the expressions below are true.

   $a.\ 2^x \cdot 2^x = 64$ $b.\ 3^x \cdot 3^x = 81$

4. Write the following expressions in a shorter way replacing product with power:

   1) $a \cdot b \cdot b \cdot b \cdot b \cdot b =$
   2) $3m \cdot m \cdot m \cdot 2k \cdot k \cdot k \cdot k =$
   3) $(ab) \cdot (ab) \cdot (ab) \cdot (ab) \cdot (ab) \cdot (ab) =$
   4) $2n \cdot 2n \cdot 2n =$
   5) $(5m)(5m) \cdot 2n \cdot 2n \cdot 2n =$
   6) $a \cdot b \cdot b \cdot b \cdot b \cdot b$

5. Write the number which extended form is written below:

   Example: $2 \cdot 10^3 + 7 \cdot 10^2 + 2 \cdot 10 + 6 = 2726;$

   $a)\ 2 \cdot 10^3 + 4 \cdot 10^2 + 5 \cdot 10 + 8;$ $b)\ 7 \cdot 10^3 + 2 \cdot 10^2 + 0 \cdot 10 + 1;$

   $c)\ 9 \cdot 10^3 + 3 \cdot 10 + 3;$ $e)\ 4 \cdot 10^3 + 1 \cdot 10^2 + 1 \cdot 10 + 4;$

6. What should be the exponent for the equation to hold?

   Example: $8^* = 512$

   Answer: $8^3 = 512$

   $a)\ 2^* = 64;$ $b)\ 3^* = 81;$ $c.)\ 7^* = 343$

7. Come up with the problem about the distance between two objects, that can be solved by the formula, and solve it.

   Example: $d = 500 - 2.5(70 + 30)$

   Problem: Two cities are 500 miles apart. A bus and a car started moving toward each other. Speed of the car is 70 m/h, speed of the bus is 30 m/h. What would be the distance between them in 2.5 hours?

   $d = 500 - 2.5(70 + 30) = 500 - 2.5 \cdot 100 = 250\ miles$

   1) $d = 18 + (16 + 4) \cdot 3$

   2) $d = 96 - 4 \cdot (56 - 40)$

8. Mother is twice as old as her daughter. Father is 5 years older than mother. Together they are 120 years old. How old is father?