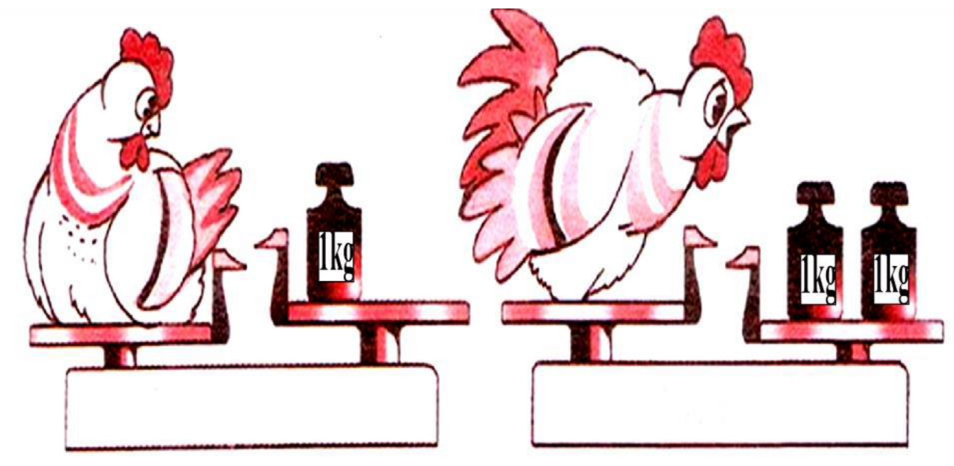
Math 4a. Classwork 13.

Decimals

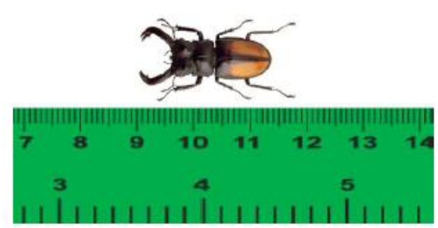
In a process of measurement, we compare a standard unit, such as 1m for length, 1kg for mass, 1degree Celsius for temperature, and so on (we can use another standard units, for example 1 foot, 1 degree Fahrenheit) with the quantity we are measuring. It is very likely that our measurement will not be exact and whole



number of standard units will be either smaller, or greater than the measured quantity. In order to carry out more accurate measurement we have to break our standard unit into smaller equal parts. We can do this in many different ways. For example, we can take $\frac{1}{2}$ of a standard unit and continue measuring. If we didn't get exact n units plus $\frac{1}{2}$ of a unit we have to subdivide further:

$$n + \frac{1}{2} + \frac{1}{2} \cdot \left(\frac{1}{2}\right) + \cdots$$

It turns out that perhaps the most convenient way is to divide a unit into 10 equal parts, then each of one tenth into another 10 even smaller equal parts and so on. In this way we will get a series of fractions with denominators 10, 100, 1000 and so on:

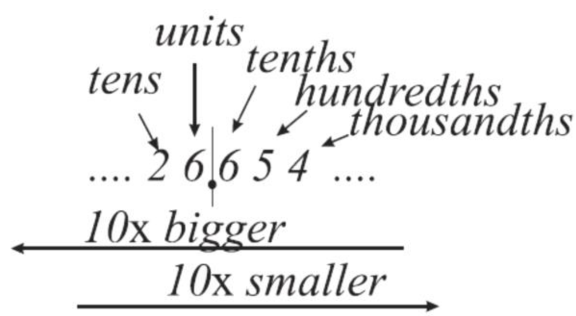


$$\frac{1}{10}, \frac{1}{100}, \frac{1}{1000} \ldots.$$

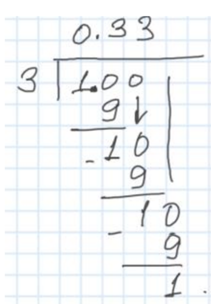
The result of our measurement can be written in a 10 based place value system.

$$26.654 = 10 \cdot 2 + 1 \cdot 6 + \frac{1}{10} \cdot 6 + \frac{1}{100} \cdot 5 + \frac{1}{1000} \cdot 4 = 10 \cdot 2 + 1 \cdot 6 + \frac{6}{10} \cdot + \frac{5}{100} + \frac{4}{1000}$$
$$= 10 \cdot 2 + 1 \cdot 6 + \frac{600}{1000} + \frac{50}{1000} + \frac{4}{1000}$$

Of course, all such numbers can be expressed in the fractional notation as fractions with denominators 10, 100, 1000 …, but in decimal notation all arithmetic operations are much easier to perform.

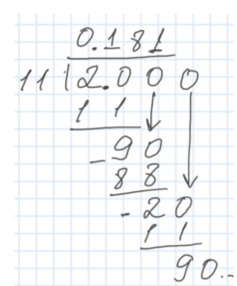


How the fraction can be represented as decimal? One way to do it, just divide numerator by denominator, as usual. For example:



$$\frac{1}{3} = 1:3 = 0.3333 \ldots = 0.\overline{3}$$

Another example,



$$\frac{2}{11} = 2:11 = 0.1818 \ldots = 0.\overline{18}$$

$$\frac{3}{5} = 3:5 = 0.6$$

Can you notice the difference? If the denominator of the fraction can be prime factorized into the product of only 2 and/or 5, fraction can be written as a fraction with denominator 10, 100, 1000 … Such fraction can be represented as a finite decimal, any other fraction will be written as infinite periodical. For now, we are going to work only with finite decimals.

Examples:

$0.3 = \frac{3}{10}$; $0.27 = \frac{2}{10} + \frac{7}{100} = \frac{27}{100}$; $0.75 = \frac{75}{100} = \frac{3 \cdot 25}{4 \cdot 25} = \frac{3}{4}$

$$\frac{1}{25} = \frac{1}{5 \cdot 5} = \frac{1}{5 \cdot 5} = \frac{1 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 2 \cdot 2} = \frac{4}{10 \cdot 10} = \frac{4}{100} = 0.04$$

$$\frac{7}{8} = \frac{7}{2 \cdot 2 \cdot 2} = \frac{7 \cdot 5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} = \frac{875}{1000} = 0.875$$

How do we perform the multiplication. If we need to multiply the natural number by 10 (or 100, or 10000):

$$245 \cdot 10 = (100 \cdot 2 + 10 \cdot 4 + 5) \cdot 10 = 100 \cdot 10 \cdot 2 + 10 \cdot 10 \cdot 4 + 10 \cdot 5$$
$$= 1000 \cdot 2 + 100 \cdot 4 + 10 \cdot 5 + 1 \cdot 0 = 2450$$

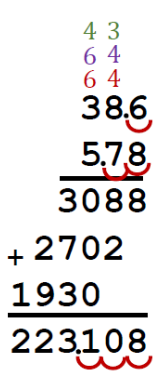
$$245 \cdot 100 = (100 \cdot 2 + 10 \cdot 4 + 5) \cdot 100 = 100 \cdot 100 \cdot 2 + 10 \cdot 100 \cdot 4 + 100 \cdot 5$$
$$= 10000 \cdot 2 + 1000 \cdot 4 + 100 \cdot 5 + 10 \cdot 0 + 1 \cdot 0 = 24500$$

Using the distributive property, we have just shown that when we need to multiply any natural number by 10 we just need to write 0 at the end of a number, increasing all place values 10 times.

If we need to multiply the decimal by 10 (or 100)

$$245.23 \cdot 10 = (100 \cdot 2 + 10 \cdot 4 + 5 + 0.1 \cdot 2 + 0.01 \cdot 3) \cdot 10$$
$$= 100 \cdot 10 \cdot 2 + 10 \cdot 10 \cdot 4 + 10 \cdot 5 + 0.1 \cdot 10 \cdot 2 + 0.01 \cdot 10 \cdot 3 =$$
$$= 1000 \cdot 2 + 100 \cdot 4 + 10 \cdot 5 + 1 \cdot 2 + 0.1 \cdot 3 = 2452.3$$

Using the distributive property, we proved that the result will be the number with decimal point moved one step to the right. (2 steps for multiplication by 100, and so on), It’s equivalent to increasing all place values 10 times.

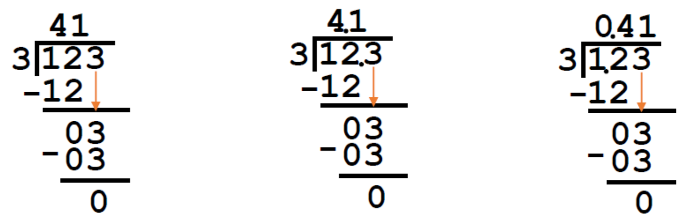


$$230:10 = 230 \cdot \frac{1}{10} = (100 \cdot 2 + 10 \cdot 3 + 1 \cdot 0) \cdot \frac{1}{10} = \frac{100}{10} \cdot 2 + \frac{10}{10} \cdot 3 + \frac{0}{10}$$
$$= 20 + 3 = 23$$

$$235:10 = 235 \cdot \frac{1}{10} = (100 \cdot 2 + 10 \cdot 3 + 1 \cdot 5) \cdot \frac{1}{10} = \frac{100}{10} \cdot 2 + \frac{10}{10} \cdot 3 + \frac{1}{10} \cdot 5$$

$$= 20 + 3 + \frac{5}{10} = 23.5$$

To perform the long multiplication of the decimals, we do the multiplication procedure as we would do with natural numbers, regardless the position of decimal points, then the decimal point should be placed on the resulting line as many steps from the right side as the *sum of decimal digits of both numbers*. When we did the multiplication, we didn't take into the consideration the fact, that we are working with decimals, it is equivalent to the multiplication of each number by 10 or 100 or 1000 … (depends of how many decimal digits it has). So, the result we got is greater by $10 \cdot 100 = 1000$ (in our example) time than the one we are looking for:



$$38.6 \cdot 5.78 = 38.6 \cdot 10 \cdot 5.78 \cdot 100 : (10 \cdot 100) = 386 \cdot 578 : 1000$$

$0.7 \cdot 10$ $: 2$ $- 0.3$ $: 0.4$ ______	$5 : 10$ $\cdot 0.2$ $+2$ $: 0.7$ ______	$4 - 0.8$ $: 0.8$ $: 10$ $\cdot 0.5$ ______	$0.9 + 0.06$ $: 0.3$ $- 0.2$ $\cdot 0.1$ ______	$1 - 0.7$ $\cdot 5$ $: 15$ $\cdot 100$ ______
$1 - 0.25$ $\cdot 2$ $: 0.3$ $- 0.05$ ______	$0.9 - 0.09$ $: 9$ $+ 0.6$ $\cdot 10$ ______	$23.9 - 3.9$ $\cdot 0.15$ $- 0.8$ $: 0.1$ ______	$12 + 0.6$ $: 3$ $- 0.2$ $\cdot 2.5$ ______	$1 - 0.4$ $\cdot 5$ $- 0.5$ $: 5$ ______

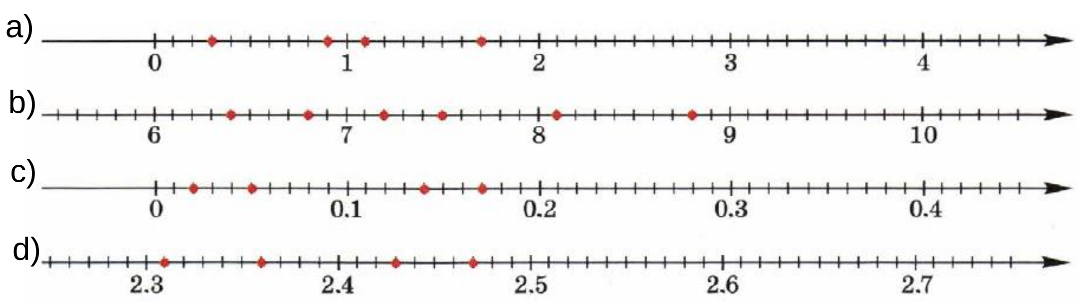
Exercises:

1. Write in decimal notation the following fractions:
   Example:

$$1\frac{3}{25} = 1 + \frac{3}{25} = 1 + \frac{3 \cdot 4}{25 \cdot 4} = 1 + \frac{12}{100} = 1.12$$

$1\frac{1}{10}$; $2\frac{4}{10}$; $4\frac{9}{10}$; $24\frac{25}{100}$; $98\frac{3}{100}$; $1\frac{1}{100}$; $4\frac{333}{1000}$; $8\frac{45}{1000}$; $75\frac{8}{10000}$; $9\frac{565}{10000}$

2. Which numbers are marked on the number lines below:



3. Evaluate:

   $a.$ $1.2 + 2.3 + 3.4 + 4.5 + 5.6 + 6.7 + 7.8;$

   $b.$ $2.3 + 3.4 + 4.5 - 5.6 + 6.7 + 7.8 + 8.5 + 9.2;$

   $c.$ $1.7 + 3.3 + 7.72 + 3.28 + 1.11 + 8.89;$

   $d.$ $18.8 + 19 + 12.2 + 11.4 + 0.6 + 11;$

4. On a graph paper draw a number line, use 10 squares as a unit. Mark points with coordinates 0.1, 0.5, 0.7, 1.2, 1.3, 1.9.

5. Which fractions below can be written in as a finite decimal:

$\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$,

$\frac{1}{11}$, $\frac{1}{12}$, $\frac{1}{13}$, $\frac{1}{14}$, $\frac{1}{15}$, $\frac{1}{16}$, $\frac{1}{17}$

Why do you think so?

6. Write decimals as fractions and evaluate the following expressions:

$a.\ \frac{2}{3}+0.5;$ $b.\ \frac{1}{3}\cdot 0.9;$ $c.\ \frac{3}{16}\cdot 0.16$

$d.\ 0.6-\frac{2;}{5}$ $e.\ 0.4{:}\frac{2}{7};$ $f.\ \frac{9}{20}:0.03$

7. Which part of 1 m is 1 cm?
   Which part of 1 km is 1 m?
   Which part of 1 cm is 1 mm?
   Which part of 1 m is 1 dm?
   Which part of 1 kg is 1 g?
   Which part of 1 g is 1 mg?

8. 1 kilogram of candies costs 16 dollars. How much

   a. 0.5 kg will cost?
   b. 1.2 kg will cost?
   c. 0.75 kg will cost?
   d. 0.4 kg will cost?
   e. 2.5 kg will cost?