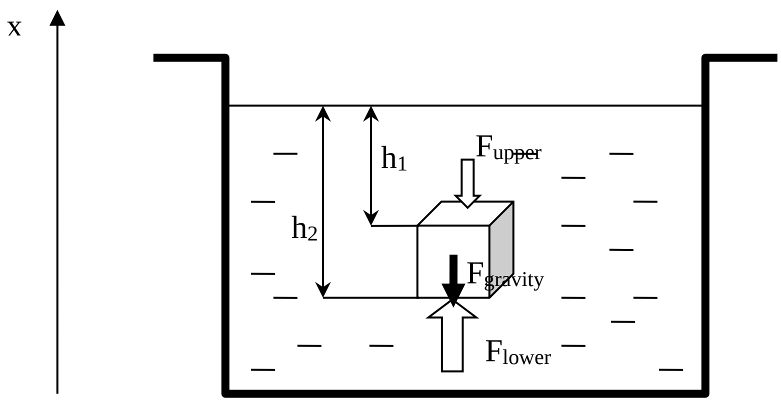
Homework 15

Buoyancy.

Let us now imagine that we have immersed a cube made from some material with the density ρ_{cube} and let it go. Will it sink or float up? To check that we have to calculate total net force applied to the cube in vertical direction.



The pressure force applied to the upper facet of the cube is:

$$F_{upper} = \rho_{liquid} \cdot g \cdot h_1 \cdot S \quad (1)$$

For the lower facet we have:

$$F_{lower} = \rho_{liquid} \cdot g \cdot h_2 \cdot S \quad (2)$$

The gravity force is

$$F_{gravity} = m_{cube} \cdot g = \rho_{cube} \cdot V \cdot g \quad (3)$$

Using the second Newton's law we can write:

$$\begin{aligned} ma &= \rho_{liquid} \cdot g \cdot h_2 \cdot S - \rho_{liquid} \cdot g \cdot h_1 \cdot S - \rho_{cube} \cdot g \cdot V \\ ma &= \rho_{liquid} \cdot g \cdot S \cdot (h_2 - h_1) - \rho_{cube} \cdot g \cdot V \end{aligned} \quad (4)$$

Here *m* is the mass of the cube, *V* is the cube's volume. But h_2-h_1 is l and $S\ l=V$. So we have:

$$ma = g \cdot V \cdot \rho_{liquid} - g \cdot V \cdot \rho_{cube} = g \cdot V \cdot (\rho_{liquid} - \rho_{cube}) \quad (5)$$

<u>Note that this equation is valid just to find the sign of acceleration! To calculate the real acceleration it is not correct, since we have not taken into account the "connected" mass of water which is replacing the space left by the cube.</u>

It is very important that we have to know <u>average</u> density of the cube. The cube can be nonuniform, have hollows inside etc. To calculate the average density we have to take the total mass of the object and divide to the volume of the object.

If the density of water is higher than the average density of the cube, then the acceleration is positive and the cube will go up – along our positive axis. This means that the cube will float.

Problems:

1. We have a silver sphere with the wall thickness of 1mm and the diameter of 30cm. Will the sphere sink if we will put it in to water? The density of silver is $10500 kg/m^3$. The density of water is $1000 kg/m^3$. (The volume of a ball is $4/3\pi R^3$, R is the radius of the ball)
2. Instead of ball you have silver cylinder with the same wall thickness. The radius of the base is 10cm, the height is 20cm. You put it vertically in the water. Will it float? If yes, find the height of the cylinder part below the water level (assuming that the stable position of the cylinder is vertical).