Homework 10.

Equation of state of the ideal gas.

Unlike simple mechanical systems, gases can not be practically characterized by coordinates and velocities of each molecule – there is just not possible to perform calculations over billions of billions of molecules. Instead we can pick up some average parameters such as temperature T, pressure P and volume V (volume here is the volume of jar or bottle where the gas is kept). Pressure is measured in N/m^2 (Pa), Volume in m^3, Temperature in degrees according to Kelvin's scale (K). One Kelvin's degree equals to 1 Celsius degree, but zero at the Celsius scale is 273 degrees at the Kelvin's scale (or simply 273K). For example, room temperature is 27^oC, but 300K.

For ideal gas (we call gas "ideal" if the molecules do not interact with each other -they do not attract or repulse) T,P and V are connected by a simple equation:

$$PV = nRT\ ,$$

where *n* is the number of molecules measured in *moles*, R is a constant which equals 8.31 J/(mole K), or $8.31x10^3$ J/(kmole K). One mole is a certain *number* of atoms or molecules. If we take $\sim 6x10^{23}$ molecules of, say, water we will have one mole of water ($6.02x10^{23}$ is called Avogadro's number). 1 kilomole (kmole) = 1000 moles.

The number is named after Amedeo Avogadro, italian physicist:

Amedeo Avogadro (1776-1856)

How to find the number of moles (or kmoles) if we know a mass of a substance? First we have to find a mass of one molecule of the substance. To do that, we need to take a look into the periodic table of elements.

Periodic table of elements

Chemical elements are the "building blocks" of nature. All the objects around us are "constructed" from chemical elements. Despite great variety of the objects and substances around us there are only 118 chemical elements (some of them are not shown in the table below). They are systematized and arranged in the table which is called periodic table of elements.

hydrogen 1 **H** 1.0079																		helium 2 **He** 4.0026
lithium 3 **Li** 6.941	beryllium 4 **Be** 9.0122												boron 5 **B** 10.811	carbon 6 **C** 12.011	nitrogen 7 **N** 14.007	oxygen 8 **O** 15.999	fluorine 9 **F** 18.998	neon 10 **Ne** 20.180
sodium 11 **Na** 22.990	magnesium 12 **Mg** 24.305												aluminium 13 **Al** 26.982	silicon 14 **Si** 28.086	phosphorus 15 **P** 30.974	sulfur 16 **S** 32.065	chlorine 17 **Cl** 35.453	argon 18 **Ar** 39.948
potassium 19 **K** 39.098	calcium 20 **Ca** 40.078		scandium 21 **Sc** 44.956	titanium 22 **Ti** 47.867	vanadium 23 **V** 50.942	chromium 24 **Cr** 51.996	manganese 25 **Mn** 54.938	iron 26 **Fe** 55.845	cobalt 27 **Co** 58.933	nickel 28 **Ni** 58.693	copper 29 **Cu** 63.546	zinc 30 **Zn** 65.39	gallium 31 **Ga** 69.723	germanium 32 **Ge** 72.61	arsenic 33 **As** 74.922	selenium 34 **Se** 78.96	bromine 35 **Br** 79.904	krypton 36 **Kr** 83.80
rubidium 37 **Rb** 85.468	strontium 38 **Sr** 87.62		yttrium 39 **Y** 88.906	zirconium 40 **Zr** 91.224	niobium 41 **Nb** 92.906	molybdenum 42 **Mo** 95.94	technetium 43 **Tc** [98]	ruthenium 44 **Ru** 101.07	rhodium 45 **Rh** 102.91	palladium 46 **Pd** 106.42	silver 47 **Ag** 107.87	cadmium 48 **Cd** 112.41	indium 49 **In** 114.82	tin 50 **Sn** 118.71	antimony 51 **Sb** 121.76	tellurium 52 **Te** 127.60	iodine 53 **I** 126.90	xenon 54 **Xe** 131.29
caesium 55 **Cs** 132.91	barium 56 **Ba** 137.33	57-70 *	lutetium 71 **Lu** 174.97	hafnium 72 **Hf** 178.49	tantalum 73 **Ta** 180.95	tungsten 74 **W** 183.84	rhenium 75 **Re** 186.21	osmium 76 **Os** 190.23	iridium 77 **Ir** 192.22	platinum 78 **Pt** 195.08	gold 79 **Au** 196.97	mercury 80 **Hg** 200.59	thallium 81 **Tl** 204.38	lead 82 **Pb** 207.2	bismuth 83 **Bi** 208.98	polonium 84 **Po** [209]	astatine 85 **At** [210]	radon 86 **Rn** [222]
francium 87 **Fr** [223]	radium 88 **Ra** [226]	89-102 **	lawrencium 103 **Lr** [262]	rutherfordium 104 **Rf** [261]	dubnium 105 **Db** [262]	seaborgium 106 **Sg** [266]	bohrium 107 **Bh** [264]	hassium 108 **Hs** [269]	meitnerium 109 **Mt** [268]	ununnilium 110 **Uun** [271]	unununium 111 **Uuu** [272]	ununbium 112 **Uub** [277]		ununquadium 114 **Uuq** [289]				

*Lanthanide series	lanthanum 57 **La** 138.91	cerium 58 **Ce** 140.12	praseodymium 59 **Pr** 140.91	neodymium 60 **Nd** 144.24	promethium 61 **Pm** [145]	samarium 62 **Sm** 150.36	europium 63 **Eu** 151.96	gadolinium 64 **Gd** 157.25	terbium 65 **Tb** 158.93	dysprosium 66 **Dy** 162.50	holmium 67 **Ho** 164.93	erbium 68 **Er** 167.26	thulium 69 **Tm** 168.93	ytterbium 70 **Yb** 173.04
**Actinide series	actinium 89 **Ac** [227]	thorium 90 **Th** 232.04	protactinium 91 **Pa** 231.04	uranium 92 **U** 238.03	neptunium 93 **Np** [237]	plutonium 94 **Pu** [244]	americium 95 **Am** [243]	curium 96 **Cm** [247]	berkelium 97 **Bk** [247]	californium 98 **Cf** [251]	einsteinium 99 **Es** [252]	fermium 100 **Fm** [257]	mendelevium 101 **Md** [258]	nobelium 102 **No** [259]

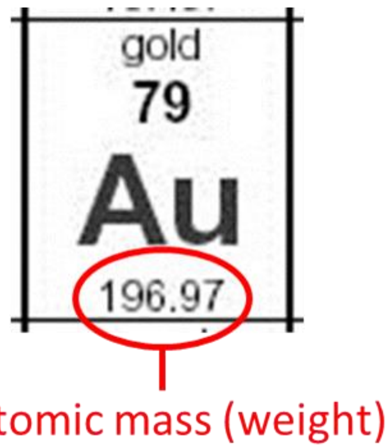
Periodic table of elements.

The periodic table was first suggested by a Russian chemist Dmitri Mendeleev in 1869. He found that if the chemical elements are arranged according their atomic mass, their chemical properties exhibit periodicity, that is why it is called "periodic".

Dmitri Mendeleev (1834-1907).

Only two of the chemical elements – mercury and bromine - are liquids at normal conditions (T=300K, atmospheric pressure), eleven elements are gases. The other elements are solids except nine elements (109-111 and 113-118) in the end of the table whose chemical properties are still unknown.

At the bottom of each cell of the table there is a number which represent the mass of the atom (atomic mass) in the so cold atomic units of mass.



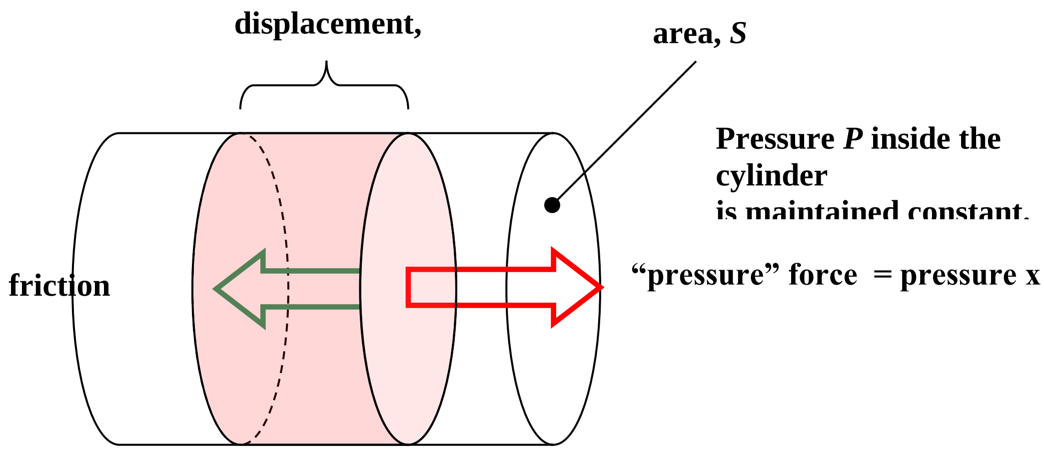
One atomic unit of mass is $1.66x10^{-24}$g, or $1.66x10^{-27}$kg. It was chosen in such a way that if we take 1 mole of a substance (that means $6.02x10^{23}$ molecules), the mass of this 1 mole will be numerically equal to the atomic mass, but in grams.

For example, atomic mass of hydrogen (H) is ~1. A molecule of hydrogen consists of 2 atoms, so the mass of the molecule is 2 atomic units of mass. If we take $6.02x10^{23}$ molecules of hydrogen (1 mole), the total mass of the gas is 2g, or 0.002kg.

Another example: how many molecules (or moles) in 100g of water? A molecule of water consists of two atoms of hydrogen and one atom of oxygen. Let's look in the periodic table. Atomic mass of hydrogen is 1, atomic mass of oxygen is 16. So the mass of a molecule of water expressed in atomic units is 18. It means that a mass of 1 mole of water is 18g (we can say that *molecular mass* of water is 18g). Now we can easily find how many moles in 100g of water: 100g:18g/mole~5.56moles. And we have total $5.56x6.02x10^{23}=33.44x10^{23}=3.34x10^{24}$.

Work, done by the gas.

We have learned is that gas can do work. Consider gas in a cylinder with a piston. We increased pressure inside the cylinder (say, connected the cylinder to a high-pressure gas bottle). The piston moves at a constant velocity since the "pressure force" is compensated by the friction force. The expanding gas performs the work and heats the cylinder through friction. Let us calculate this work:



Work = force x displacement= pressure x area x displacement.

Or in a short form:

$$W = PSd.$$

But, as we can see, area multiplied by the displacement gives us change in volume, which we denote as ΔV. This change in volume is represented as the pink cylinder in the figure. So,

$$W = P \cdot \Delta V$$

It is interesting that this formula is valid for a cylinder of any shape as long as the pressure is *maintained constant*. If the gas is just expanding in a cylinder, the pressure changes as the gas pushes the piston outside and the work cannot be calculated that simply.

Problems:

1. There is a cylinder with a piston. The mass of the piston is 100kg, its area is $100cm^2$. The cylinder contains 28g of nitrogen at T_1=273K. The cylinder is heated up to T_2=373K. How does the piston position change? The atmospheric pressure is ~101,000 Pa.

2. How much hydrogen (in grams) is in a cylinder with a piston if it performs work of 400J being heated from 250K to 680K? The gas pressure was maintained constant. Neglect the weight of the piston.

3. There is a closed from both sides cylinder with a piston inside. The piston divides the inner volume of the cylinder to two parts. One part contains 3g of hydrogen, the other contains 17g of nitrogen. What is the ratio of the volume of "hydrogen part" to the total volume of the cylinder?