

Homework 9

States of matter.

We have started discussing states of matter. There are a lot of states of matter, but for the beginning we will talk about 3 of them. These are

1. **Solid state** (a solid object has both definite volume and shape).
2. **Liquid state** (liquids have definite volume – they are almost incompressible, but have not a shape).
3. **Gas state** (both volume and shape are determined by the container).

Gases.

Gases “work hard” for us in our everyday life – for example, hot gas expanding in the cylinder of the engine makes the car move. To design a machine which uses gas to produce work we have to know in detail the behavior of gas at different conditions. As we learned, gas consists of huge number of microscopic particles called molecules or atoms (depending on the kind of the gas). It is not possible to track down or describe the motion of each molecule in a gas volume. Fortunately, we do not have to do that. We just have to know three important parameters and the way how they depend on each other. The parameters are:

- Pressure
- Volume
- Temperature

Strictly speaking, we just have to know any two of them – then we can calculate the third.

Pressure.

From our everyday experience we know well that the result of application of force depends not only on the force magnitude and direction but also on the area to which the force is applied. For example, it is very hard to push, say, a match into a wooden panel, but we can easily do that with a pushpin. This is because in the last case the pressure produced by the pushpin is much higher. What is pressure?

In real world a force cannot be applied to an infinitely small point – the force is always applied to a certain finite area. Even the needle tip has an effective area (very small, though). To find pressure we have to divide normal force by the area to which the force is applied. So we can say that the pressure is a normal force applied to unit area.

$$\text{Pressure} = \text{Force}/\text{Area}$$

$$P=F/S$$

Pressure is measured in N/m^2 . This unit is called Pascal (Pa) after a famous French mathematician, physicist and philosopher Blaise Pascal.



Blaise Pascal (1623-1662) (www.wikipedia.org)

Gas always applies pressure to the walls of the cylinder or any other vessel. The pressure can be low as in a balloon or high as in a barrel of a gun during the shot. Our atmosphere also produces pressure. It is $\sim 100\text{kPa}$ ($101,300\text{Pa}$ to be exact) at sea level.

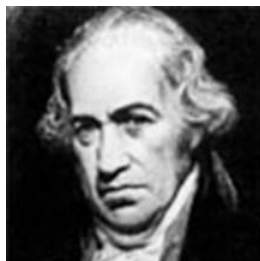
Temperature.

We know very well that a hot object has higher temperature than a cold one. So, the higher temperature of *something* the hotter that *something* is. But it is much more difficult to explain what the temperature *is*. What is the physical meaning of this parameter? We know that all of the objects around us consist of small particles which move chaotically all the time. Temperature of the object is proportional to the average kinetic energy of the particles (atoms or molecules) of the object.

If we are cooling down, say, a glass of water, molecules of water are slowing down and losing their kinetic energy (they transfer it to another object – an ice cube, for example). Since there is minimal possible kinetic energy – 0.0J (all the particles stop) we can conclude that there is the minimal possible temperature. In the real world there is not possible to stop the particles completely and cool the object down to absolute zero temperature. There are four major temperature scales.

1. Fahrenheit temperature scale
2. Celsius temperature scale
3. Kelvin temperature scale
4. Rankin scale

The property of certain objects to expand or contract depending on temperature was known since ancient times. But first person who produced thermometer with reliable operation and reproducible readings was Daniel Gabriel Fahrenheit – German physicist and engineer. In 1724 he introduced new temperature scale which is now known as Fahrenheit scale.



Daniel Gabriel Fahrenheit (1686-1736)

Fahrenheit used mercury as a working substance in his thermometer and, what is most important, he suggested the way to calibrate the device. For calibration he used three points. One was the temperature of the mix contained water, ice and special salt - ammonium chloride. An interesting property of this mixture (which is called *frigorific* mixture) is that after being prepared it reaches equilibrium at the temperature which is almost independent on the initial temperatures of the mixture components. Fahrenheit ascribed to this temperature the magnitude of 0 degrees. The other two points were temperatures of water mixed with ice (32°F) and the temperature of a human body (96 °F). 2



Anders Celsius (1701-1744)

Zero point of the Celsius scale, named after Swedish astronomer Anders Celsius (1701-1744) corresponds to the temperature of melting ice, temperature of hundred degrees corresponds to the temperature of boiling water. To recalculate Fahrenheit temperature into the Celsius temperature we have:

1. Take the temperature in Fahrenheit degrees and subtract 32.
2. Multiply the result by 5 and divide by 9 – you have the Celsius temperature.

The Kelvin scale is the scale used in physics. One degree of the Kelvin scale is equal to 1 Celsius degree, but 0oC correspond to 273 degrees at the Kelvin scale (we write 273K). And the last one, Rankin scale has same degree as Fahrenheit scale, but zero point corresponds to absolute zero (Kelvin zero).

The ideal gas laws.

There are 3 simple laws which establish the connection between temperature, pressure and volume of ideal gas. Speaking about “ideal gas” we mean the gas consisting of the particles (atoms or molecules) which do not interact (repel or attract) with each other. This is not true for most of the real gases, but if the temperature is high enough the effect of the interaction is small and real gas behaves like the ideal one. So, the laws are:

1. Boyle -Mariotte law:

Pressure x Volume = does not change, or $PV = \text{const}$

This means that if the temperature of the gas remains unchanged decreasing the gas volume we will increase the gas pressure and vice versa.

2. Charle's law:

$\frac{\text{Volume}}{\text{Temperature}}$ does not change, or $\frac{V}{T} = \text{const}$

If the pressure of the gas remains unchanged, as the temperature of the gas increases the volume of the gas increases as well. This law describes thermal expansion of gas at the constant pressure.

3. Gay-Lussac's law

$\frac{\text{Pressure}}{\text{Temperature}}$ does not change, or $\frac{P}{T} = \text{const}$

If the volume of the gas remains unchanged, increasing the gas temperature we will increase the gas pressure and vice versa.

Important note: pay special attention to the units. **Kelvin scale should be used to express temperature(!).**

The combined gas law.

All three ideal gas laws can be conveniently written in one expression:

$$\left(\frac{P \cdot V}{T}\right) = \text{const}$$

This expression is called “combined ideal gas law”. Whatever we do to the ideal gas, its pressure multiplied by volume and divided by temperature stays the same if we do not change the mass (or the number of molecules) of the gas.

Example:

A closed 1m³ cylinder with a piston contains gas at the pressure of 10000Pa and temperature 300K. Find the pressure after we heat the gas to 400K and increase the volume of the gas to 1.5m³ by moving the piston.

Solution:

We use our “universal” ideal gas law:

$$\begin{aligned} \left(\frac{P \cdot V}{T}\right)_{\text{before}} &= \left(\frac{P \cdot V}{T}\right)_{\text{after}} \\ \frac{10000Pa \cdot 1m^3}{300K} &= \frac{P \cdot 1.5m^3}{400K} \\ P &= \frac{10000Pa \cdot 1m^3 \cdot 400K}{300K \cdot 1.5m^3} \approx 8889Pa \end{aligned}$$

Problems:

1. Temperature in the room is increased from 15°C to 39°C. How many times did the average kinetic energy of the air molecules increase?
2. You slightly press two balloons of different diameter (but with the same pressure inside) against each other. Describe the shape of the balloon wall at the place of

contact: will it be flat or bent toward one of the balloons? This is a little bit challenging but interesting problem. Try to explain your answer using the combined gas law.

3. We have vertical cylinder with a piston of area A . The cylinder is filled with gas, which occupies volume V under the piston. The piston has mass m and can move without friction. What happens to the gas volume if we will move the cylinder vertically with acceleration a ? Assume that you know the atmospheric pressure P_0 and gas temperature is kept constant.