

# Ampere's Law

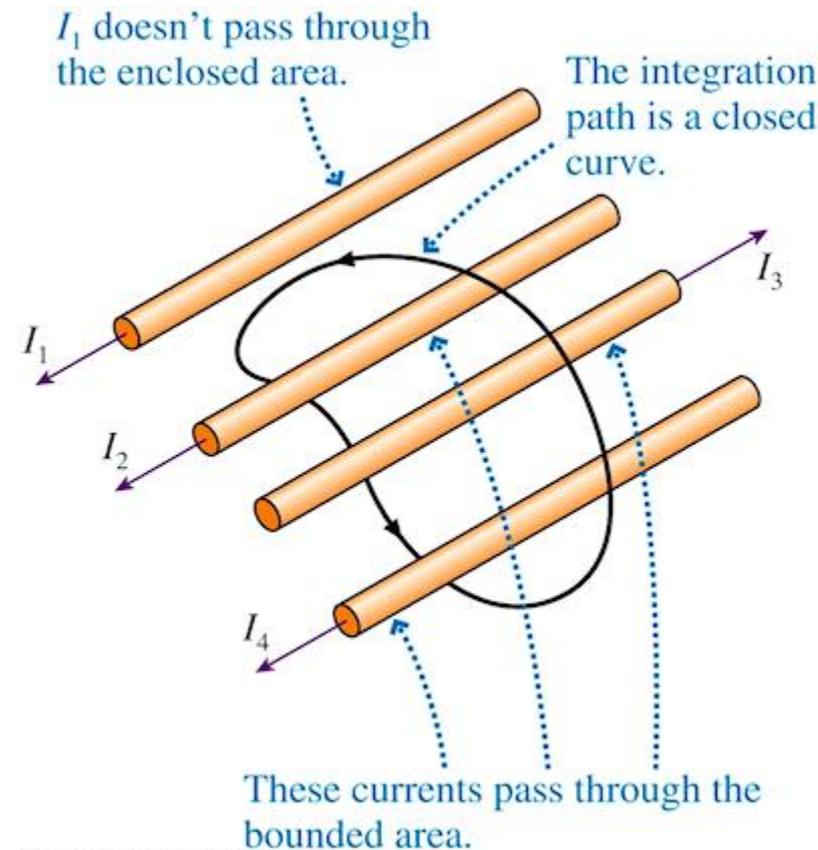
Consider an arbitrary closed loop (for instance, a circle). Ampere's Law states that the integral of magnetic field along that loop is proportional to the total current enclosed by it:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum_{\text{insidetheloop}} I$$

Note that the integral contains "dot" product that depends on the angle between vector  $\vec{B}$  and the local direction of the integration path:

$$\vec{B} \cdot d\vec{l} = |\vec{B}| |d\vec{l}| \cos \alpha$$

$$\mu_0 = 4\pi \cdot 10^{-7} T \cdot m / A$$



# Using Ampere's Law: Infinite Wire

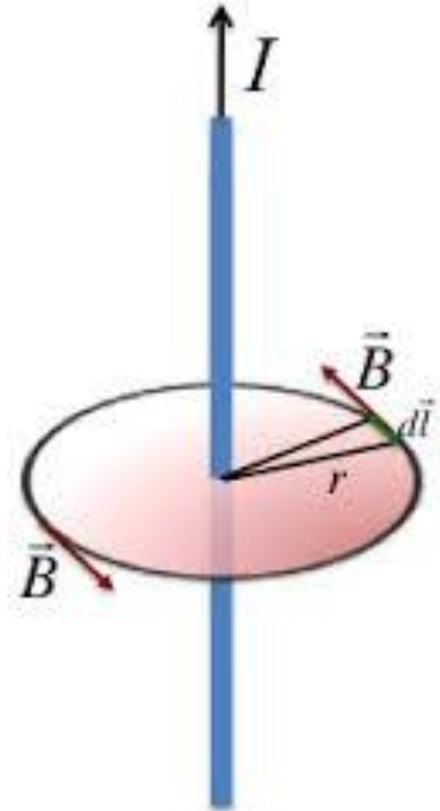
Consider a straight infinite wire carrying current  $I$ . As an integration loop we choose a circle of radius  $r$  around the wire. At any point of the loop,  $B$  is constant and directed along the path, therefore  $\cos(\alpha)=1$ .

$$\oint \vec{B} \cdot d\vec{l} = 2\pi r B$$

By using Ampere's Law, we obtain:

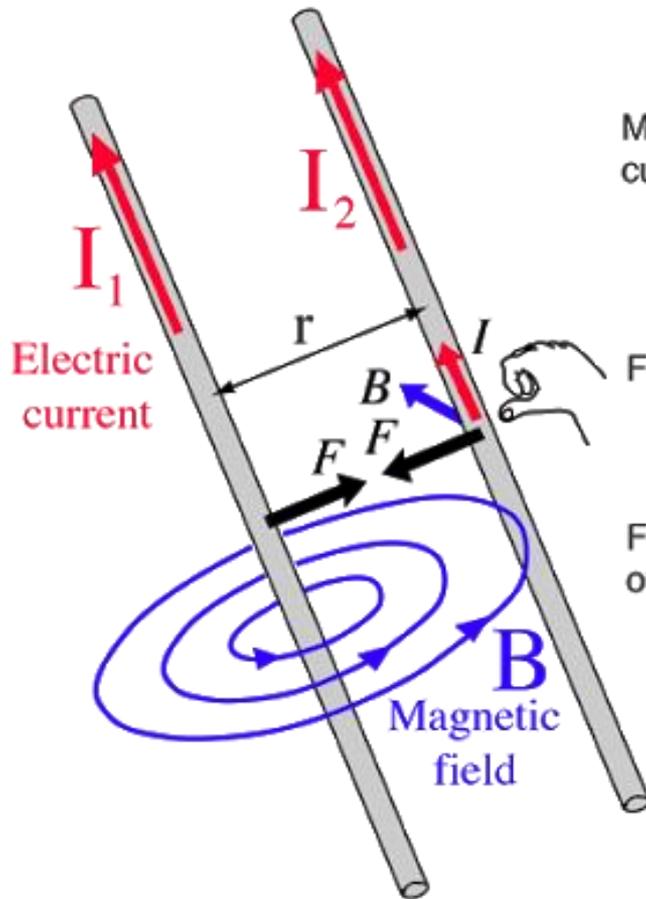
$$B = \frac{\mu_0 I}{2\pi r}$$

Direction of  $B$  is determined by the right hand rule.



# Magnetic Force Between Wires

We combine Ampere's Law with Lorentz Force,  $F=I\Delta LB$ :



Magnetic field at wire 2 from current in wire 1:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

Force on a length  $\Delta L$  of wire 2:

$$F = I_2 \Delta L B$$

Force per unit length in terms of the currents:

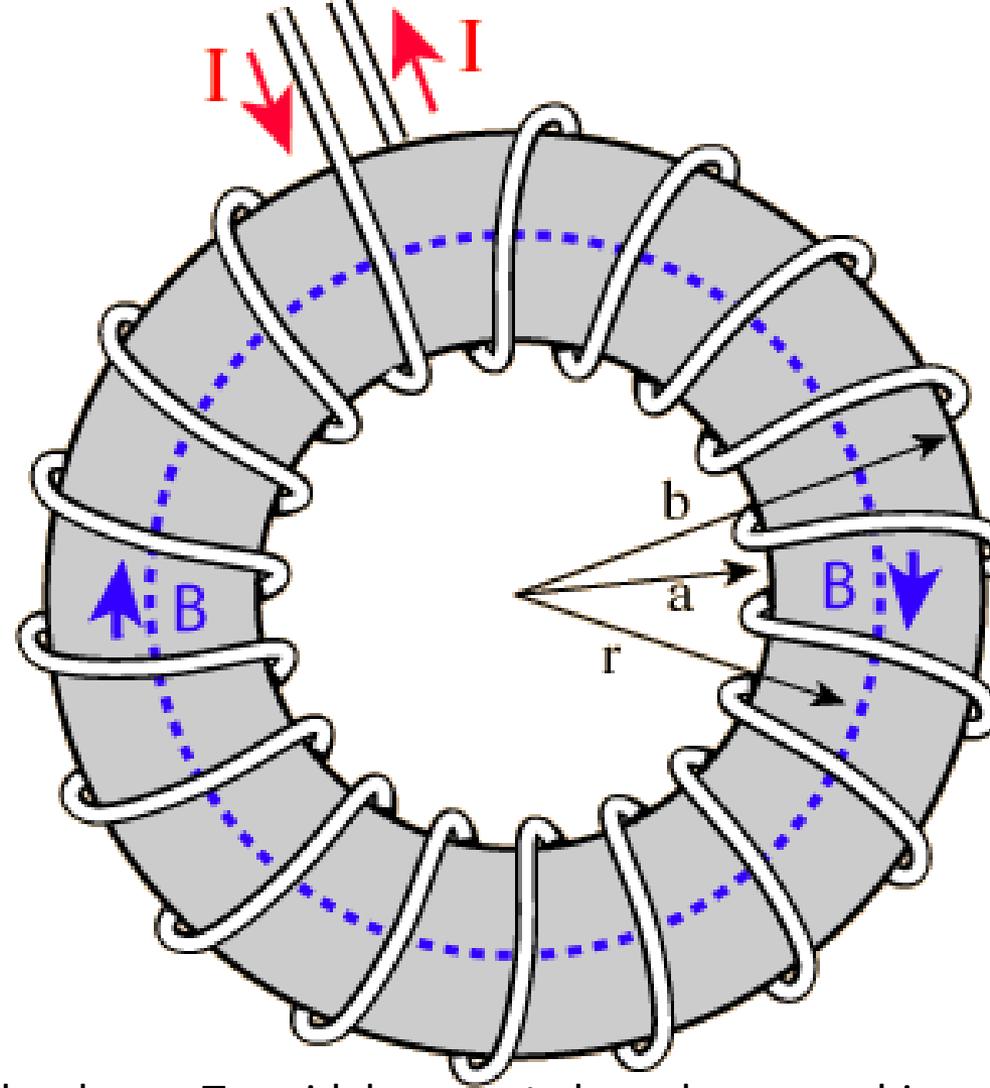
$$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

# Homework

## Problem 1

Two parallel wires of radius  $r=0.1 \text{ mm}$  each, are placed right next to each other (i. e. distance between their centers is  $2r$ ). The same current  $I$  is run through each wire. Find the value of  $I$ , at which the magnetic force between the wires is equal to the weight of each of them. Density of copper is  $9000 \text{ kg/m}^3$ .

# Homework



## Problem 2

Torus is a mathematical term for a bagel- like shape. Torroidal magnets have been used in tape recorders, and other devices. Find the magnetic field  $B$  inside of the toroidal magnet, near its centerline that has a shape of a circle of radius  $r$  (shown in the Figure as blue dashed line)). The wire makes  $N$  turns around the torus, and the current is  $I$ .

How many turns do you need to produce 1 T magnetic field in a torus of radius  $r=30$  cm, if current is  $I=1$ A?