Homework 5

Electric potential. Superposition.

Eletric potential.

The potential energy where of two charges separated by a distance *r* is

$$P = k\frac{q_1 \cdot q_2}{r} \quad (1)$$

Let us keep one of the charges, say, $\boldsymbol{q_1}$ fixed and change the charge $\boldsymbol{q_2}$. Since there is a product of the charge magnitudes in the numerator of formula (1), the potential energy will increase or decrease proportionally to the charge magnitude of $\boldsymbol{q_2}$. We can now calculate the potential energy *per unit charge*. For this we will divide the potential energy of the interacting charges $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$ by the magnitude of $\boldsymbol{q_2}$:

$$\frac{P}{q_2} = k\frac{q_1 \cdot q_2}{r} \div q_2 = k\frac{q_1}{r} \quad (2)$$

We can imagine that each point of space around the charge q_1 can be characterized by the potential energy of a positive unit charge in this point. The electrostatic potential energy of a positive unit charge in a certain point is called "***electric potential***" in this point. The electric potential is a scalar. The electric potential $\boldsymbol{\varphi}$ created by the point charge $\boldsymbol{Q}$ is:

$$\boldsymbol{\varphi = k\frac{Q}{r}} \quad (3)$$

There are 2 important issues. First: if the charge q is negative, the potential will be negative as well. Second: potential, created by a point charge is "spherically symmetrical". This means that only the distance from the charge to the point where we are measuring potential matters (rather than the exact position of the point).

The formula (3) means that a unit positive point charge placed at the distance r from the charge q will have potential energy $\boldsymbol{\varphi}$. If we will place an arbitrary charge Q at the distance r (instead of a unit charge) then the potential energy of the charge Q can be calculated as:

$$\boldsymbol{P = k\frac{Q}{r} \cdot q = \varphi \cdot q} \quad (4)$$

As we can see from the formula (3) the potential created by a point charge depends on the distance to the point charge. Difference of potentials taken in points A and B equals to the difference of potential energy of a unit positive charge in these points. Now let us look at Figure 1. Let us assume that the position of positive charge $\boldsymbol{Q}$ is fixed, and another small object having charge $\boldsymbol{q}$ is placed in a point separated from $\boldsymbol{Q}$ with a distance $\boldsymbol{R_1}$. Charge $\boldsymbol{q}$ is repelled by charge *Q*, and, being released, starts moving from charge $\boldsymbol{Q}$. When it reaches point B, separated from $\boldsymbol{Q}$ with a distance $\boldsymbol{R_2}$, it has nonzero kinetic energy, but its potential energy is less now. The gain in kinetic energy is equal to the loss of the potential one due to energy conservation.

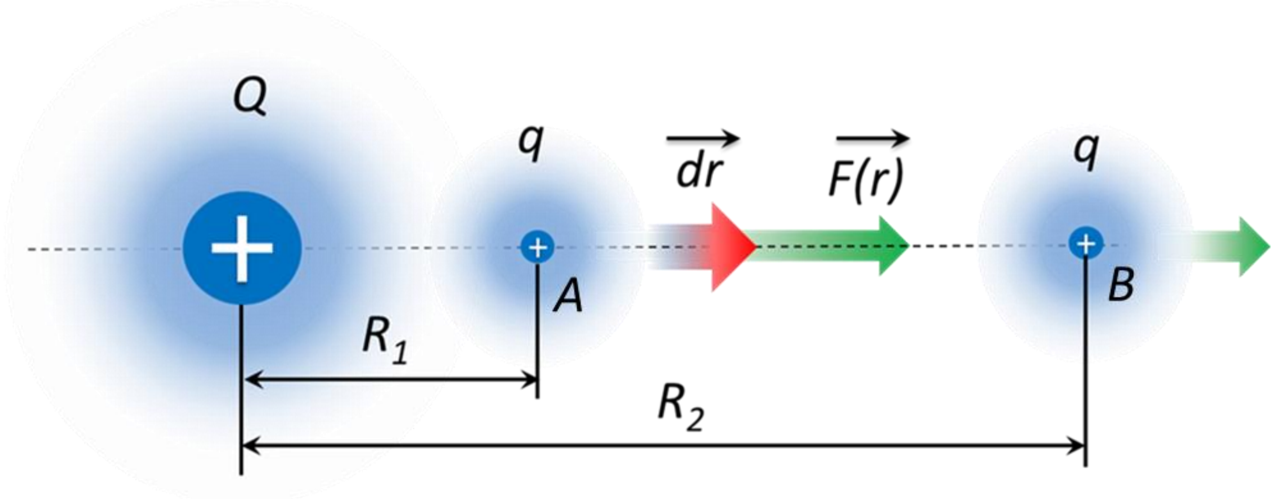


Figure 1. Work of the electric field. dr is a displacement in the direction AB, F(r) is the Coulomb force which depends on r.

But, the gain in kinetic energy is equal to the work $\boldsymbol{W_{AB}}$, done by electrostatic force on charge q. Typically, by "change" we mean "final value minus initial value" Change in potential energy then will be negative since its final valuer at $\boldsymbol{R_2}$ is less than its initial value at $\boldsymbol{R_1}$ (since $\boldsymbol{R_2}$ is larger than $\boldsymbol{R_1}$). Let us introduce a slightly different parameter: potential energy in the *initial* point minus potential energy in the *final* point. We have:

$$\boldsymbol{W_{AB} = P_A - P_B = q\varphi_A - q\varphi_B = q(\varphi_A - \varphi_B) = qU_{AB}} \qquad (5)$$

Here P_A, P_B –electrostatic potential energies in points A and B; $\boldsymbol{\varphi_A}$, $\boldsymbol{\varphi_B}$ – the electrostatic potentials, U_{AB} =$\boldsymbol{\varphi_A - \varphi_B}$ is the potential difference which is also called "***voltage drop*** *between points A and B"*, or just "***voltage*** *between points A and B"*.

We assumed that the charge was moved from point A to point B along a straight line. We may ask: "may be there is an optimal path, so if we use this path the loss of potential energy when the charge moved from one point to the other will be minimal". This does not work for electrostatic field. **The potential energy (or just potential) difference between two points does not depend on the path we choose!**

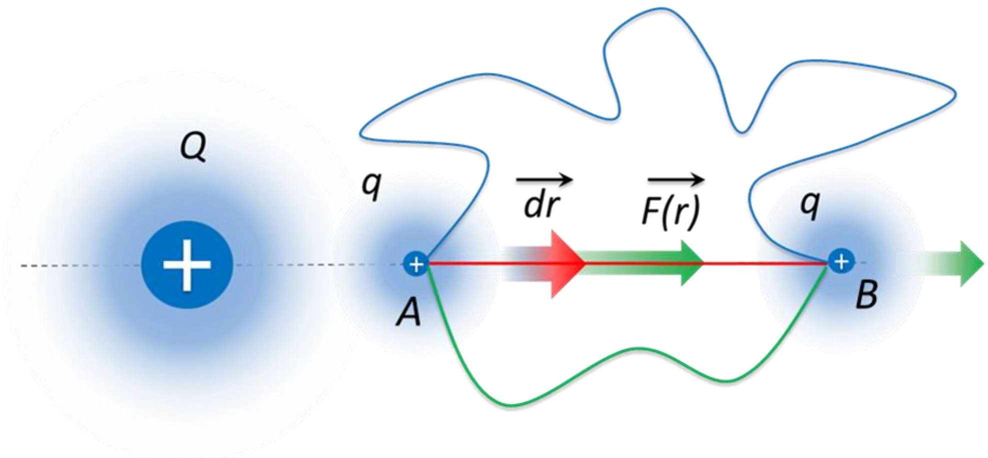
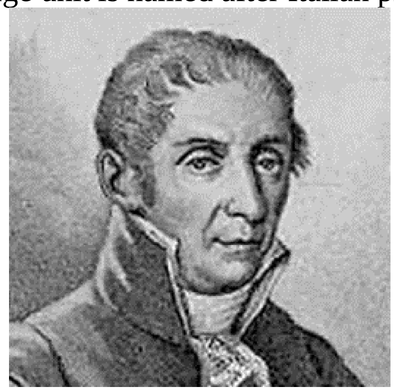


Figure 2. Potential energy difference between points A and B does not depend on the path geometry.

We can choose the green path (Figure 2) or the blue pass the potential energy difference will be the same. Let us assume for a moment that the change in potential energy depends on the path you choose and for the blue path it is, say, higher that for the green path. Then we could choose green path to go from A to B and blue pass to go from B to A and return to the same point A having higher potential energy than we initially had in this point. But this does not agree with the expression for electrostatic potential energy we obtained earlier. According to this expression if the distance is the same so is the potential energy. The electrostatic potential and voltage are measured in Volts.
1Volt=1Joule/1Coulomb. The voltage unit is named after Italian physicist Alessandro Volta:



Alessandro Volta
1745-1827

Superposition principle.

The beauty and convenience of the concept of electric potential is that using the electrical potential we can easily calculate the potential energy of a charged object in the electric field created by arbitrary configuration of other charged objects.

Let us consider the following example.

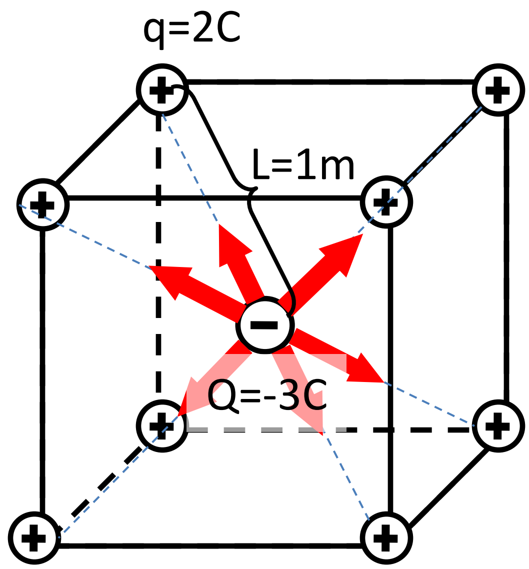


Figure 1. The red arrows show the electrostatic forces applied to the negative charge.

Given:

Eight point charges q=2C each are placed in the corners (vertices) of a cube (see Figure 1 above). The positions of the positive charges are fixed. The distance between the center and a corner of the cube is 1m. A negative charge of Q=3C is placed in the center of the cube.

Find electrostatic potential energy of the negative charge.

Solution:

As we remember, a possible way to calculate the electrostatic potential energy of a charge in a certain point is to calculate the electrostatic potential in this point and multiply it by the charge. Let us calculate the electrostatic potential in the center of the cube (black point in the Figure 2.

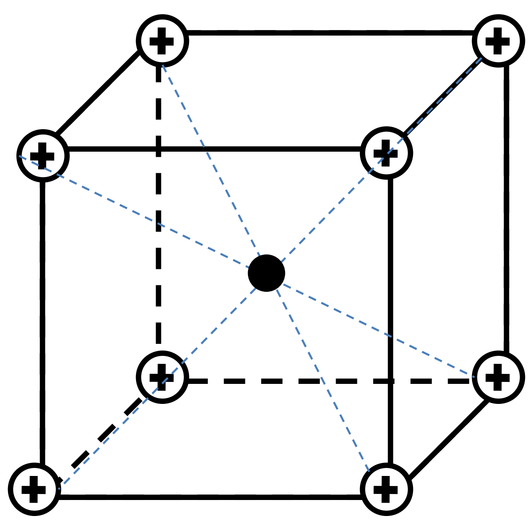


Figure 2.

At a first glance the problem looks difficult. But, in fact, it is not. To solve it, we will use the ***principle of superposition.*** According to this principle, we can calculate the potentials created in the center of the cube by each of the positive charges separately. After that we will just add these potentials together.

a) Let us pick just one positive charge (Figure 3).

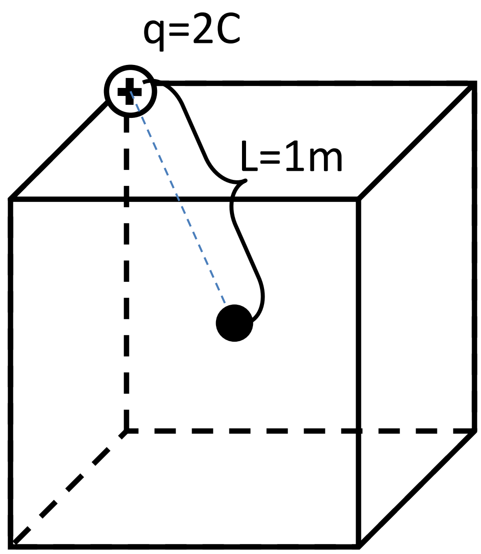


Figure 3.

Then let us calculate the electrostatic potential created by this charge at the center of the cube.

$$\boldsymbol{\varphi_{one\ charge}} = \boldsymbol{k\frac{q}{L}} \approx 8.9 \cdot 10^9 \left(\frac{N \cdot m^2}{C^2}\right) \cdot \frac{2(C)}{1(m)} \approx 1.78 \cdot 10^8 (V)$$

b) Now, we have to take another positive charge, calculate its contribution to the potential etc. But, there is a simpler way. We can use the ***symmetry principle*** which we discussed earlier. As long as all the corners of the cube occupy equivalent positions with respect to the center of the cube, there is no reason to prefer one corner to another. Thus, the contributions of the equal positive charges placed in the corners of the cube to the potential in the cube's center should be equal. So we can just multiply the contribution of one positive charge by 8 – the number of corners.

$$\boldsymbol{\varphi_{total}} = \boldsymbol{\varphi_{one\ charge}} \cdot 8 = 1.78 \cdot 10^8 (V) \cdot 8 \approx 1.42 \cdot 10^7\ (V)$$

c) Now we can easily calculate the potential energy ***P*** of the charge ***Q=-3C*** placed in the center of the cube:

$$\boldsymbol{P} = \boldsymbol{\varphi_{total}} \cdot \boldsymbol{Q} = 1.42 \cdot 10^7\ (V) \cdot (-3)(C) = -4.26 \cdot 10^7 J$$

What happens if the charges at the "bottom" vertices are negative (Figure 3)?

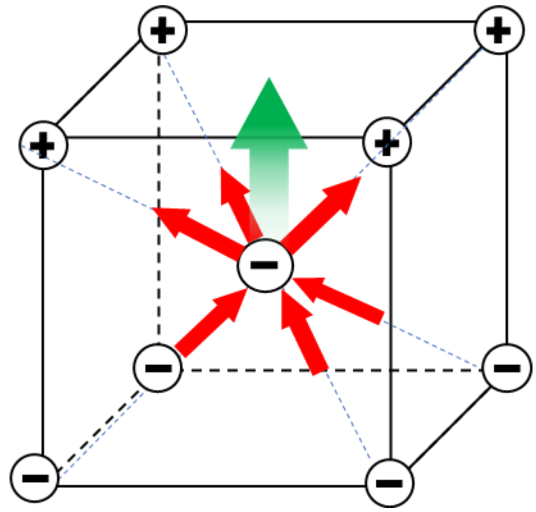


Figure 3.

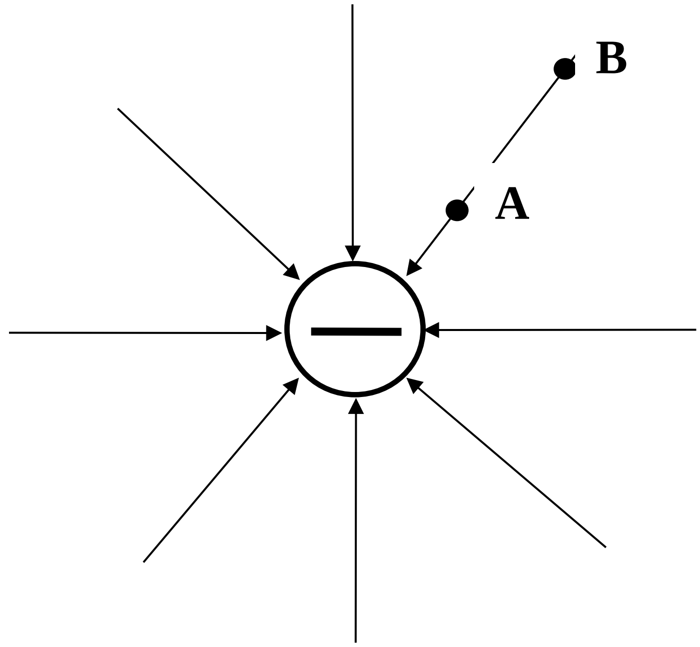
Now, as we can clearly see the negative charge in the center will be pushed up, since it is repelled from the bottom and attracted to the top. So, if we let it go, it will be accelerated and, when reaches, say, the middle of the top facet it will have some kinetic energy. Based on this, we can conclude that our charge has some potential energy while placed in the center of the cube. But let us calculate this potential energy using our recipe: calculate the potential and multiply it by the charge (-3C). The contribution of each of the top positive charges we have calculated earlier: 1.78×10^8 V. We have to multiply it by 4, since there are 4 charges on the top vertices). The contribution of each of the bottom negative charges is negative: -1.78×10^8 V. We have to multiply it by 4 as well and add the positive contribution of the top charges. The result is zero! And it is correct! But if we multiply -3C by zero to find the potential energy we will get zero. No potential energy!

To resolve this paradox let us remember that not the absolute potential energy, but the change of potential energy is physically meaningful. Potential energy in the center is zero, but it is not ***minimal*** potential energy. If you will calculate the potential energy of our charge in the center of the upper facet, you will have a negative value. So, change of the potential energy will be positive and will be equal to the kinetic energy acquired by our charge.

Problems:

1. An object with a charge of 0.01C being accelerated by electrostatic force moves from point A to point B and gains kinetic energy of 6J. Find the potential difference between points A and B.

2. There is a point charge of -1C (see picture below). The distance between the charge and the point A is 100m, the distance between the points A and B is also 100m. Find the potential difference between points A and B.



3. Find potential in the center of a uniformly charged thin sphere having a total charge of 1C and radius 1m.