COMBINED GAS LAW.

APRIL 27, 2023

Theory recap

Combined gas law. We have discussed three gas laws and in every one of them there is one parameter that we keep fixed. But what if we want to change all three parameters of the gas? Say we start from gas having pressure p_1, volume V_1 and temperature T_1. We engage the gas in some process so that as the result it has pressure p_2, volume V_2 and temperature T_2. Our question is, are these 6 parameters related to each other somehow? The answer is yes!

We can derive the relation between the aforementioned 6 parameters from the gas laws we have already established. But we have to be clever about it since our gas laws would only work if one of the three parameters is fixed. Luckily, nothing prevents us from reaching the final state in two steps: one at fixed temperature and another at fixed volume (see figure 1). We will do the following to the gas: first, we keep temperature fixed and change volume from V_1 to V_2. Pressure then also changes to some p_3 and as we know from Boyle's law:

$$p_3 V_2 = p_1 V_1 \implies p_3 = \frac{p_1 V_1}{V_2}.$$

After we are done with this, we could keep the volume fixed and now change temperature from T_1 to T_2. Pressure also changes from p_3 to p_2. As we know from Gay-Lussac's law:

$$\frac{p_2}{T_2} = \frac{p_3}{T_1} \implies \frac{p_2}{T_2} = \frac{p_1 V_1}{V_2 T_1} \implies \frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1}.$$

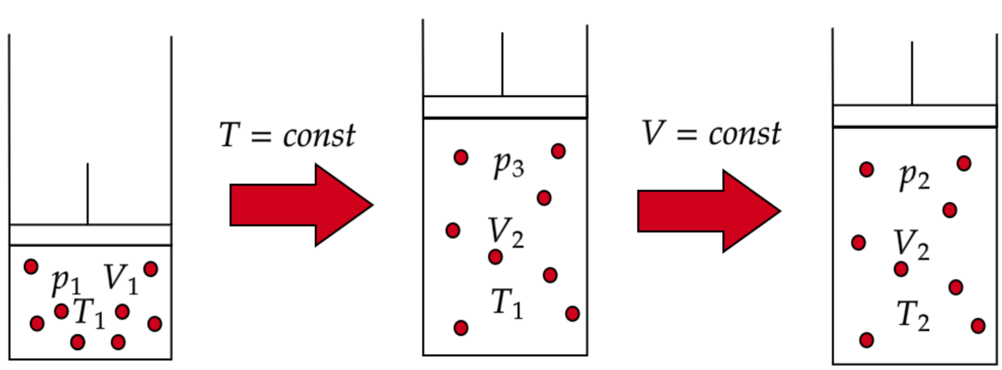


Figure 1. We could reach the final state p_2, V_2, T_2 in two steps : first keep temperature constant and then keep volume constant.

In the second step we have used the expression for p_3 derived earlier from Boyle's law.

We have derived the combined gas law. We see that the product $\frac{pV}{T}$ is the same in initial and final state. Analogously we could show that it stays the same in any two states of the gas. So, it is a constant in any process for a given gas:

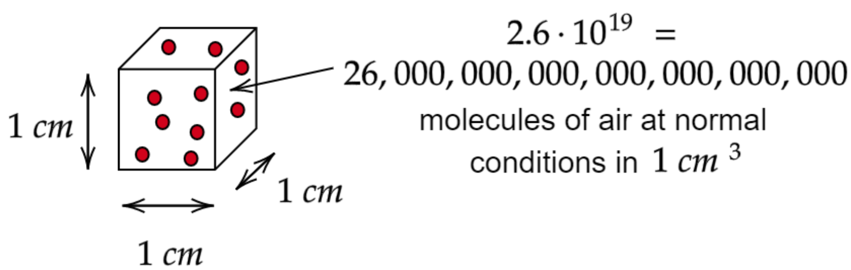
$$\frac{pV}{T} = const.$$

This law contains all three gas laws what we previously discussed. Note that we only used two laws (Boyle's law and Gay-Lussac's law) in derivation and as a result we see the third one (Charles' law) as well. We have already seen this - Charles' law can be derived from the other two (in fact, if you start with any two of the three laws, the third one would follow).

The value of the combined gas law is that if we know all three parameters in a state 1 of the gas and we know two parameters in a state 2, we can find the remaining parameter in state 2.

But what is the constant in the combined gas law equal to? It should be proportional to the amount of molecules of the gas. Why? Imagine that we consider two containers with gas, each having the same pressure and temperature but one being two times bigger in volume. Then in each half of the bigger container we should have the same amount of molecules as in the smaller container. Overall we have twice as many molecules in the big container then. $\frac{pV}{T}$ in the large container is also two times larger than in the small container (volume is two times bigger), so we see the relation between the constant in the combined gas law and the number of molecules.

Amount of molecules in gas. This brings us to the question: how many molecules are there in a gas? This of course depends on volume, as well as pressure and temperature. Let us take air at normal conditions - atmospheric pressure $p = 10^5$ Pa and temperature $T = 273$K (which is the same as 0° C) and consider a little cube with side 1 cm. The volume of this cube is 1 cm^3 and it has approximately $2.6 \cdot 10^{19}$ air molecules. How much is $2.6 \cdot 10^{19}$? If we write it out, it is 26,000,000,000,000,000,000 but it is still hard to appreciate how big this number is.



Let me put it this way: imagine an ant and the planet Earth. What if we tried to cover the entire surface of Earth (including the oceans) completely with a single layer of ants? How many ants would we need? This amount is actually of the same order of 10^{19} as the number of molecules in a single 1 cm^3 of air!

Amount of substance. Because there are so many molecules in ordinary objects around us it would not be very convenient to use the actual number of molecules in all computations. So people came up with a convention: for every substance we say that we have 1 mole of this substance if we have $6.02 \cdot 10^{23}$ molecules of this substance. This number is called the Avogadro number or the Avogadro constant:

$$N_A = 6.02 \cdot 10^{23}$$

If we have twice the Avogadro number of molecules we have two moles, and so on. The physical quantity measured in moles is called **amount of substance** and denoted by letter n. Mathematically it is defined like this:

$$n = \frac{N}{N_A} \text{ moles},$$

where N is number of molecules in the substance and N_A is the Avogadro number.

Equation of state of ideal gas. Summing up, combined gas law is:

$$\frac{pV}{T} = nR$$

n is the amount of substance in moles. Everything in this formula is already familiar to us except R. R is called gas constant (or universal gas constant) and it is equal to

$$R = 8.31 \ \frac{\text{J}}{\text{mole} \cdot \text{K}}$$

R is another fundamental physical constant which we meet in our course. It describes the behavior of any gas, no matter its chemical composition.

Usually the last equation is written in the following equivalent form, which is a bit easier to remember because there are no fractions:

$$pV = nRT$$

This equation is very important. It is called the **equation of state of ideal gas**. It contains combined gas law and through it the three individual gas laws that we discussed, but actually is contains more than that. Equation of state of ideal gas tells us that all gases, no matter what atoms or molecules they consist of, have the same relation between four basic parameters: pressure, volume, temperature and amount of substance. So if you take the same amount of molecules of hydrogen or water vapor or chlorine, you wound not be able to distinguish them by the pressure they produce at given volume and temperature. If they have the same volume and temperature, they will have the same pressure. This is the great universality which is only found among gases because of their physical simplicity: in an ideal gas approximation they are just a bunch of molecules flying around without interacting with each other.

As an example of this universality let us calculate the volume that 1 mole of any gas would take at normal conditions: normal atmospheric pressure $p_0 = 101.3$ kPa and temperature $T = 0°$ C $= 273$ K. From the equation of state of ideal gas we get

$$V = \frac{nRT}{p_0} = \frac{1 \cdot 8.31 \cdot 273}{101,300} \text{m}^3 = 0.0224 \text{ m}^3 = 22.4 \text{ L}$$

This number is useful to keep in mind, just to understand the scale of how much is one mole.

Homework

1. Temperature of a gas is increased 2 times (measured in Kelvin) and its' volume is decreased 3 times. How does the pressure change?

2. What is the volume occupied by 3 moles of an ideal gas at the temperature of 57°C and pressure 150 kPa?

3. We have vertical cylinder with a piston of area A. The cylinder is filled with gas, which occupies volume V under the piston. The piston has mass m and can move without friction. What will happen to the gas volume if we move the cylinder vertically with acceleration a? Assume that you know the atmospheric pressure P_0 and gas temperature is kept constant.

***4.** A balloon probe is filled with gas at temperature $t_1 = 27$°C up to pressure 105 kPa. After rising to height where the outside pressure is $p_0 = 80$ kPa the volume of the balloon increased by $n = 5\%$. The pressure in the balloon is higher than outside by $\Delta p = 5$ kPa. Find the temperature of outside air at this height assuming the gas inside the balloon has the same temperature as the outside air.