DISPLACEMENT AT MOTION WITH ACCELERATION

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Theory recap

Recall from the previous class that accelerated motion occurs whenever velocity changes. If velocity changes at a constant rate, this means that acceleration is constant and velocity ($\vec{v}$) dependence on time (t) is very simple:

$$\vec{v}(t) = \vec{v}_0 + \vec{a}\ t$$

Here $\vec{v}_0$ is initial velocity and $\vec{a}$ is acceleration.

The next thing we would like to know is how far do we travel when accelerating constantly. We considered an application of this question: if we want to explore a vertical cave but do not see the bottom and are not sure that our rope will be of enough length. To find out depth of the cave we could throw a pebble there and measure the time it takes to reach the bottom (which we would hear). It would be smart to actually "throw" the pebble with zero initial velocity, simply let it go from our hand. Then we need to find how to relate the distance traveled by the pebble to time of travel.

If speed was constant, we would know the answer for distance right away: speed multiplied by time. The problem is that speed is changing. But motion with constant acceleration has a nice feature: speed changes at a constant rate. Because of this, average velocity is an algebraic average of initial velocity and final velocity. For a pebble initially at rest (velocity equals 0) after time t velocity will be gt (we choose the positive direction to be down, it will be more convenient). Algebraic average of initial and final velocity is then

$$v_{avg} = \frac{0 + gt}{2} = \frac{gt}{2}$$

Displacement is equal to average velocity multiplied by time:

$$d = v_{avg}t = \frac{gt}{2}t = \frac{gt^2}{2}$$

Remember that this formula applies for motion with constant acceleration starting at zero velocity. If it took our pebble 3 seconds to fall, we calculate the depth of the cave to be

$$h = \frac{gt^2}{2} = \frac{10 \cdot 3^2}{2}\text{m} = 45\ \text{m}$$

So a standard 50 m rope fortunately will be enough.

In general we could have started started with some non-zero initial velocity $\vec{v_0}$. Let us use the same idea of calculating average velocity. Now we will think of some general accelerated motion, not necessarily free fall, and denote acceleration by $\vec{a}$. After time t velocity will be $\vec{v_0} + \vec{a}t$ (see our general formula at the beginning of this page), so average velocity is

$$\vec{v}_{avg} = \frac{\vec{v_0} + \vec{v_0} + \vec{a}t}{2} = \vec{v_0} + \frac{\vec{a}t}{2}$$

Then displacement is

$$\vec{d} = \vec{v}_{avg}t = \vec{v_0}t + \frac{\vec{a}t}{2}t = \vec{v_0}t + \frac{\vec{a}t^2}{2}$$

Homework

1. A soldier shoots vertically up. The bullet starts moving up at a speed of 400m/s. In what time the bullet will stop?

2. Let us find how long the runway in an airport should be so that an airplane has enough space to gain speed. An airplane initially at rest accelerates with constant acceleration $a = 2\ m/s^2$ until it gets to the takeoff speed of $v_t = 80\ m/s$.

(a) What time does it take the airplane to reach the takeoff speed?

(b) How far does the airplane move before taking off? (normally you would want a runway to be about twice as long - to allow some space for braking in case of emergency).

(c) What is average velocity of the airplane during acceleration?

3. A coin is falling down for 3 seconds. Initial velocity of the coin is 0. Find the displacement of the coin during the third second.