

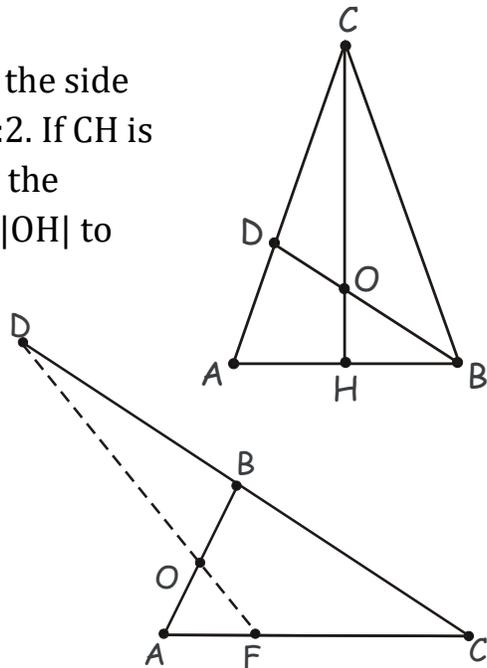
Homework for November 13, 2022

## Geometry.

Review the classwork handout. Solve the unsolved problems from previous homeworks. Try solving the following problems, some of which we solved in the past using the similarity of triangles and Thales theorem, now using the method of point masses and the Law of Lever.

### Problems.

1. Prove that if a polygon has several axes of symmetry, they are all concurrent (cross at the same point).
2. Prove that medians of a triangle divide one another in the ratio 2:1, in other words, the medians of a triangle “trisect” one another (Coxeter, Gretzer, p.8).
3. In isosceles triangle  $ABC$  point  $D$  divides the side  $AC$  into segments such that  $|AD|:|CD|=1:2$ . If  $CH$  is the altitude of the triangle and point  $O$  is the intersection of  $CH$  and  $BD$ , find the ratio  $|OH|$  to  $|CH|$ .
4. Point  $D$  belongs to the continuation of side  $CB$  of the triangle  $ABC$  such that  $|BD| = |BC|$ . Point  $F$  belongs to side  $AC$ , and  $|FC| = 3|AF|$ . Segment  $DF$  intercepts side  $AB$  at point  $O$ . Find the ratio  $|AO|:|OB|$ .



# Algebra.

Review the classwork handout and complete the exercises. Solve the remaining problems from the previous homework (you may skip the ones considered in class). Solve the following problems.

1. Using Euclid's algorithm, provide the continued fraction representation for the following numbers. Using the calculator, compare the values obtained by truncating the continued fraction at 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>, ... level with the value of the number itself (in decimal representation).

a.  $\frac{1351}{780}$

b.  $\frac{25344}{8069}$

c.  $\frac{29376}{9347}$

d.  $\frac{6732}{1785}$

e.  $\frac{2187}{2048}$

f.  $\frac{3125}{2401}$

2. Is there a number,  $x$ , represented by the following infinite continued fraction? If so, find it.

a.  $x = 5 - \frac{6}{5 - \frac{6}{5 - \frac{6}{5 - \dots}}}$

b.  $x = 2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \dots}}}$

c.  $x = 1 - \frac{6}{1 - \frac{6}{1 - \frac{6}{1 - \dots}}}$

3. Write the first few terms in the following sequence ( $n \geq 1$ ),

$$n \text{ fractions } \left\{ \begin{array}{l} \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}} \\ \dots + \frac{1}{1+x} \end{array} \right. = ?$$

- a. Try guessing the general formula of this fraction for any  $n$ .  
 b. Using mathematical induction, try proving the formula you guessed.

4. Can you prove that,

a.

$$\frac{3+\sqrt{17}}{2} = 3 + \frac{2}{3 + \frac{2}{3 + \frac{2}{3 + \dots}}} ?$$

b.  $1 = 3 - \frac{2}{3 - \frac{2}{3 - \frac{2}{3 - \dots}}} ?$

c.

$$\frac{4}{2 + \frac{4}{2 + \frac{4}{2 + \dots}}} = 1 + \frac{1}{4 + \frac{1}{4 + \frac{1}{4 + \dots}}} ?$$

Find these numbers?