MATH 8: HANDOUT 1
REVIEW I

1. Open parentheses and expand the following expressions
   (a) $(a+b)^2 =$
   (b) $(a-b)^3 =$
2. Factor the following expressions:
   (a) $a^2 - b^2 =$
   (b) $a^3 - b^3 =$
   (c) $a^3 + b^3 =$
3. Expand as sums of powers of x:
$$(2x+1)^2(2-3x)$$
4. A group of 19 people want to select a chairperson and two associates. How many ways there are for them to do so?
5. Solve the equation
$$x + \frac{1}{x} = 4.25$$
6. Consider the following quadratic equation:
$$x^2 - 5x - 14 = 0$$
   (a) What is the discriminant of this equation?
   (b) Sketch a graph of this quadratic polynomial
   (c) Solve the equation.
7. Let $x + y = 7$ and $xy = 8$
   (a) Write down the quadratic equation so that x and y are its solutions.
   (b) Calculate $x^2 + y^2$.

Find the sum of the first 10 terms for the series: 4, 7, 10, 13, . . .

Arithmetic Series. Recall how do you obtain your formula. Don't just memorize.

$$S = a_1 + a_2 + a_3 + \cdots + a_n = n \times \frac{a_1 + a_n}{2}$$

Proof: we write the sum in 2 ways, in increasing order and in decreasing order:

$S = a_1 + a_2 + a_3 + \cdots + a_n$
$S = a_n + a_{n-1} + a_{n-2} + \cdots + a_1$

Adding up left and right sides:
$$2S = (a_1 + a_n) + (a_2 + a_{n-1}) + (a_3 + a_{n-2}) + \cdots$$
We notice that:
$$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \cdots$$
$$2S = (a_1 + a_n) \times n$$
$$S = \frac{(a_1 + a_n) \times n}{2}$$

$5 + 20 + 80 + \ldots + 20480$ can be written as,

$5 \times 1 + 5 \times 4^1 + 5 \times 4^2 + \ldots + 5 \times 4^6$

$$= 5 \times \left(1 + 4 + 4^2 + \ldots + 4^6\right) = 5 \times \left(\frac{4^7 - 1}{4 - 1}\right) = 27305.$$

Geometric Series. Recall how do you obtain your formula. Don't just memorize.

$$a_1 + a_2 + a_3 + \cdots + a_n = a_1 \times \frac{(1 - q^n)}{1 - q}$$

<u>Proof:</u> To prove this, we write the sum and we multiply it by q:

$$S = a_1 + a_2 + a_3 + \cdots a_{n-1} + a_n$$
$$qS = qa_1 + qa_2 + qa_3 + \cdots + qa_{n-1} + qa_n$$

Remember that $qa_{n-1} = a_n$, so that the last term is $qa_n = q \times (a_1 \times q^{n-1}) = a_1 \times q^n$:

$$S = a_1 + a_2 + a_3 + \cdots a_{n-1} + a_n$$
$$qS = a_2 + a_3 + a_4 + \cdots + a_n + a_{n+1}$$

Subtracting the second equality from the first, and canceling out the terms, we get:

$$S_n - qS_n = (a_1 - a_{n+1}), \quad \text{or}$$

$$S_n(1 - q) = (a_1 - a_1 q^n)$$

$$S_n(1 - q) = a_1(1 - q^n)$$

from which we get the formula above.