Homework 18: Inequalities, quadratic functions in vertex form

HW18 is Due February 22; submit it to Google classroom 15 minutes before the class time.

1. Converting from standard to vertex form

We know how to draw the graph of $y = x^2$. It's a parabola.

- We know that the graph of $y = x^2 + k$ can be obtained from the graph of $y = x^2$ by shifting up by k units (or down, if k < 0)
- We know that the graph of $y = (x + h)^2$ can be obtained from the graph of $y = x^2$ by shifting left by h units (or right, if h < 0).
- Based on the two facts above, we can draw a graph of any function of the type $y = (x + h)^2 + k$. The values $(-h, k)$ are the coordinates of the vertex of the parabola.

In general. we can transform any quadratic function $\boldsymbol{y = ax^2 + bx + c}$ to $\boldsymbol{y = a(x + h)^2 + k}$.
This is called transforming from standard into vertex form. The coefficient a has the same value in both forms. Note that, if $b = 0$, then your equation in standard form is already in a vertex form with vertex coordinates $(0, k = c)$.

We will use two ways to convert a quadratic function from standard into vertex form:

- **Method 1: completing the square**. We have learned how to do this using the formulas for fast multiplication.
  Example: $y = 2x^2 + 4x - 2 = 2[\boldsymbol{x^2 + 2x} - 1] = 2\left[x^2 + 2.x.1 + 1^2 - 1^2 - 1\right] = 2[(x+1)^2 - 1 - 1] = 2[(x+1)^2 - 2] = 2(x+1)^2 - 4.$

- **Method 2: find the vertex**. Determine the coefficients a, b, c. Find the vertex *x-coordinate* $\boldsymbol{x_v = -h = -\frac{b}{2a}}$. Then, substitute $\boldsymbol{x_v}$ in the function you are converting and solve for y, $y = a\boldsymbol{x_v}^2 + b\boldsymbol{x_v} + c$. The found value is the vertex *y-coordinate,* $\boldsymbol{y_v = k}$. Write the function's vertex form $a(x+h)^2 + k = y$.
  Example: $y = 2x^2 + 4x - 2, \quad a = 2,\ b = 4,\ c = -2$

  Vertex *x-coordinate:* $x_v = -h = -\frac{b}{2a} = -\frac{4}{2.2} = -1$ **But** $h = 1$!
  Vertex *y-coordinate:* $y_v = 2{x_v}^2 + 4x_v - 2 = 2(-1)^2 + 4(-1) - 2 = 2 - 4 - 2 = -4, \quad k = -4$

  New function: to $y = a(x + \boldsymbol{h})^2 + \boldsymbol{k} = 2(x+1)^2 - 4$

Homework problems

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Solve the following quadratic inequalities:
   a. $2x^2 - 3x + 1 > 0$ (Convert into factored form, see your class notes!)
   b. $4(x - \sqrt{3})(x - 4) \leq 0$
   c. $\frac{x}{2x-3} \geq \frac{1-x}{2x-3}$.

2. For each part a), b), c), d), e), and f), graph all functions in desmos.com (you could graph more than one function at a time). Each part studies a different parameter (For example, in a) only the parameter a≠ 0, the other two are set to zero, b = 0, c = 0). You do not need to attach the graphs, but you may ***sketch*** the graphs for each part on the same coordinate system!

   a. Study parameter $a \neq 0$: $y = x^2$, $y = 2x^2$, $y = 5x^2$
   b. Study parameter $a \neq 0$: $y = x^2$, $y = -x^2$, $y = -2x^2$
   c. Study parameter $a \neq 0$: $y = x^2$, $y = \frac{2}{3}x^2$, $y = \frac{1}{3}x^2$
   d. Study parameter $c \neq 0$ (also $a \neq 0$): $y = x^2 + 5$, $y = x^2 - 3$
   e. Study parameter $b \neq 0$ (also $a \neq 0$): $y = x^2 - 4x$, $y = x^2 + 4x$
   f. Study parameter $c \neq 0$ (also $a \neq 0$, $b \neq 0$): $y = x^2 - 4x + 5$, $y = x^2 + 4x + 5$

   At the end of this problem, **explain the meaning of each parameter** a, b, c in the quadratic function when the function is written in a standard form: $y = ax^2 + bx + c$.
   Please use your own words and full sentences to explain what you have discovered about each parameter.

3. Convert the quadratic functions from standard into vertex form. Please state which method(s) you are using: Method 1 or Method 2, as shown on page 1 here or in your class notes.

   a) $y = x^2 - 5x + 5$
   b) $y = x^2 - 4x + 2$
   c) $y = x^2 - x - 1$
   d) $y = -x^2 + 3x - 0.5$
   e) $y = x^2 + 4x - 45$