MATH 7: HANDOUT 2
ALGEBRAIC EXPRESSIONS AND IDENTITIES

MAIN ALGEBRAIC IDENTITIES

Reminder from previous class:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$a^2 - b^2 = (a-b)(a+b)$$
$$(ab)^n = a^n b^n$$
$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

There are also formulas for the third power (cube) of a sum and a difference:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

and also for the sum and the difference of two cubes (but not for the sum of two squares!)

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Note that one can factor $(a-b)$ in terms of $\sqrt{a}$ and $\sqrt{b}$, to get

$$a - b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$$

which is very helpful in simplifying expressions with roots, for example:

$$\frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2} - 1$$

We also discussed solving simple equations where the left hand side is factored as product of linear factors, such as $(x-2)(x+3) = 0$. Since any of the factors can make the left hand side to be zero, we record this mathematically in form **either of**

$$(x-2)(x+3) = 0 \quad \Leftrightarrow \quad \left[\begin{array}{l} x-2 = 0 \\ x+3 = 0 \end{array} \right.$$

Note that the arrow $\Leftrightarrow$ points both ways meaning *equivalence*, so we have the same solutions on the left and the right.

CLASSWORK

1. Expand as sums of powers of x and y:

(a) $(2x+5)^3$

(b) $(2-4x)^3$

(c) $(2x+3)(4x^2-6x+9)$

(d) $(x-2y)(x^2+2y+4y^2)$

2. Factor (i.e., write as a product) the following expressions:

(a) $a^3 + 6a^2b + 12ab^2 + 8b^3$

(b) $27x^3 - 27x^2y + 9xy^2 - y^3$

(c) $27x^3 - 125$

(d) $a^3 + 64b^3$

(e) $a^6 - b^6$

3. Solve the following equations.

(a) $(x-2)(x+5)=0$
(b) $(y^2-9)(y+5)=0$
(c) $x^2-7x=0$
(d) $x^3-27=0$

4. Simplify expressions

(a) $\frac{1}{3-\sqrt{2}}$
(b) $\frac{\sqrt{2}+3}{\sqrt{2}-1}$
(c) $\frac{\sqrt{5}+1}{\sqrt{5}-1}$
(d) $\frac{\sqrt{6}+2}{\sqrt{3}-1}$

HOMEWORK

1. Collect the factors in the following expressions (simplify to the form $2^r3^sa^mb^n$):

(a) $8a^3b^2(27a^3)(2^5ab)$ (b) $3^2(2ab)^3(16a^2b^5)(24b^2a)$ (c) $16a^2b^3(6ab^4)(ab^2)^3$

2. Expand as sums of powers of x and y:

(a) $(y-2x)^3$
(b) $(4x+3)^3$
(c) $(1-3x)(1+6x+9x^2)$
(d) $(x+5y)(x^2-5y+25y^2)$

3. Factor (i.e., write as a product) the following expressions:

(a) $3a^2-12a+12$
(b) $a^2-b^2-10b-25$
(c) $(x-2)^2-(y+3)^2$
(d) $(x-2)^2-10(x-2)+25$
(e) $64-a^6b^6$
(f) $3x^3-x^2y+6x^2y-2xy^2+3xy^2-y^3$

4. Solve the following equations.

(a) $5(x+1)=3x+2$
(b) $(x-3)(x+4)=0$
(c) $(x^2-1)(x+2)=0$
(d) $x^2+4x=0$
(e) $x^3+4x=0$
(f) $y^4-8y=0$

5. Simplify expressions

(a) $\frac{1}{2-\sqrt{3}}$
(b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$
(c) $\frac{\sqrt{5}+2}{\sqrt{5}-1}$
(d) $\frac{\sqrt{7}}{\sqrt{7}-2}$

6. Amanda has an average of 92 on her seven tests. What should she get on her $8th$ test to have an average of 93?