**MATH 7: HANDOUT 3**
**ALGEBRAIC EXPRESSIONS AND IDENTITIES**

## Exponents Laws

If $a$ is a real number, $n$ is a positive integer, we define $a^n = \underbrace{a \times a \times \cdots \times a}_{n\text{-times}}$

$$\begin{aligned}
&a^0 = 1\\
&a^m \times a^n = a^{m+n}\\
&a^m \div a^n = a^{m-n}\\
&(ab)^n = a^n b^n\\
&\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\\
&a^{-n} = \frac{1}{a^n}\\
&(a^m)^n = a^{mn}
\end{aligned}$$

Why do we assume that $a^0 = 1$? We can see it from the following argument:

$$a^1 = a^{1+0} = a^1 a^0$$

Therefore, we must have the following equality (if we want to maintain the properties of exponents – and we do!):

$$a = a \cdot a^0$$
$$a^0 = 1$$

Now, why do we assume that $a^{-n} = \frac{1}{a^n}$? We can see it from the following argument:

$$a^{-n} \cdot a^n = a^{-n+n} = a^0 = 1.$$

Therefore, dividing both parts of the equation by $a^n$, we get:

$$a^{-n} = \frac{1}{a^n}.$$

## Radicals

Now, what should the fractional powers be? Let us figure out what $a^{1/2}$ is. We will use similar logic as above:

$$a^{1/2} \cdot a^{1/2} = a^1 = a.$$

Therefore, $(a^{1/2})^2 = a$; from here, we can see that $a^{1/2} = \sqrt{a}$. Similarly, we can find that $a^{1/n} = \sqrt[n]{a}$. In general, we have the following properties:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}, n \neq 0$$
$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

## Main Algebraic Identities

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$a^2 - b^2 = (a-b)(a+b)$$

All of these formulas can be proven by performing multiplication, e.g. $(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$. You can verify the others by yourself.

## Classwork

**1.** Calculate

(a) $\sqrt[3]{54}$

(b) $8^{2/3} \div 9^{3/2}$

(c) $\sqrt[6]{(x^2y^3)^4}$

(d) $\sqrt[3]{a^2b} \div \sqrt{ab^2}$

**2.** Without a calculator, find

(a) $101^2$

(b) $998^2$

(c) $995 * 1005$

(d) $303 * 198$

## Homework

Please try to do as many of the problems below as you can. Some of these problems are similar to those we have discussed in class; some are new. You solutions should include explanations and not just an answer.

**1.** Without a calculator, compute

$$19999 \cdot 20001$$

Is there a shorter way of doing it than the straightforward multiplication?

**2.** Expand

(a) $2x(a + 2b + 3c)$

(b) $-3y(a - ay + by)$

(c) $(a^2 + 2a + 1)(a + 1)$

(d) $(b^2 - 2b + 1)(b - 1)$

(e) $(4x - 7y)(4x + 7y)$

(f) $(6x^2 - y)(7x^2 - 2x - 5)$

**3.** Factor (i.e., write as a product) the following expressions:

(a) $x^2 + 3x^3$

(b) $x^2 - 2x - yx + 2y$

(c) $4x^2 - 4x + 1$

(d) $4x^2 + 16x + 2xy + 8y$

(e) $a^2 + 4ab + 4b^2$

(f) $a^4 - b^4$ [Hint: $a^4 = (a^2)^2$. ]

**4.** John takes 15 min to walk from school to the bus station. Jim takes 20 min to walk from the school to the bus station. If the difference in their speeds is 2 km/h, how far is the station from the school?

**5.** Simplify:

(a) $\dfrac{1}{x+1} - \dfrac{1}{x-1}$ (b) $\left(1 + \dfrac{1}{x}\right) \div (x+1)$ (c) $\left(1 + \dfrac{1}{x}\right) \div \left(1 - \dfrac{1}{x}\right)$