REVIEW 1

- Basics of mathematical logic ("Knights and Knaves"). AND($A \wedge B$), OR($A \vee B$), NOT($\overline{A}$), IF($A \implies B$, "implies"). Truth tables.

    1. On the Island of Knights(kt) and Knaves(kv), a traveler meets Sue and Jack. Sue says that Jack is a knave, while Jack says "I and Sue are knights". Who is a knight and who is a knave?

    2. On the Island of Knights(kt) and Knaves(kv), Bill says that Carl is a knave. Carl tells you, "Of me and Dan, only one is a knight", and Dan says, "Bozo and I are different". Who is a knight and who is a knave?

    3. Show that saying "If A is True, then B is False" is equivalent to saying "If B is True, then A is False', whatever statements A and B are.

    4. Show that saying "If A is True, then B is True" and "If B is False, then A is True" implies that B has to be True.

- Sets: union($A \cup B$), intersection($A \cap B$), complement($\overline{A}$). Cardinality.

    1. Let
       F = set of students who know French,
       G = set of students who know German,
       R = set of students who know Russian.
       Describe the following sets:
       (a) $F \cup G$;   (b) $F \cup (G \cap R)$;   (c) $(F \cap G) \cup (F \cap R)$;   (d) $(F \cap \overline{G})$.

    2. In a class of 25 students, 10 students know French, 5 students know Russian, 12 know neither. How many students know both French and Russian?

    3. Let $A = \{\forall x \mid 1 \le x \le 4\}$, $B = \{\forall x \mid x \ge 2\}$, $C = \{\forall x \mid x \le 3\}$. Write and draw the following sets:
       (a) $\overline{A}$;   (b) $A \cap B$;   (c) $A \cap \overline{C}$;   (d) $A \cap (\overline{B \cap C}$;

- Factorials and permutations

    1. How many ways can one rearrange 7 books on a shelf? What if there are three identical ones?

    2. What are the odds to pull four red cards in a row out of a 52-deck card?

    3. What are the odds that at least two students in our class have birthdays on the same day? In the same week? (assume that there are 52 weeks in a year)

- Arithmetic and geometric progressions (sequences). Formulas for the terms and the sums.

    1. What is the 12-th term of an arithmetic sequence a_n, if $a_3 = 8$ and $a_9 = 44$?

    2. An arithmetic sequence of 100 numbers starts with $a_1 = 10$ and ends with $a_{100} = 200$. What is the sum of all the terms from a_1 to a_{100}?

    3. What are the first and the fifth terms of a geometric sequence a_1, 24, 36, 54, a_5?

    4. In a geometric series of 15 terms, $a_1 = 5$ and $a_{15} = 320$, what is a_8?

    5. Compute the sum $1 + \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^8}$.

    6. Find the sum of the infinite series $\frac{1}{9} + \frac{1}{27} + \frac{1}{81} \ldots$.