MATH 7: HOMEWORK 22
Invariants, and asymptotes
April 3, 2022

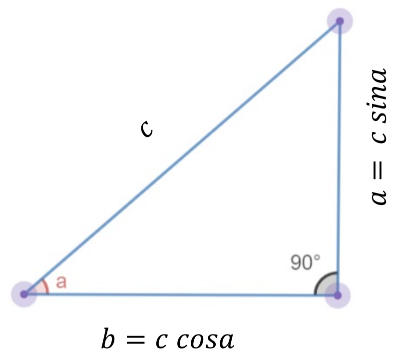
1. Definition for sin and cos of an angle

As we discussed, for any angle α, we can find invarians : (sine) $sin\alpha$ and (cosine) $cos\alpha$

In general, for a right-angle triangle with hypothenuse not equal to 1, the ***sina*** and ***cosa*** of the angle are defined as:

$$sina = \frac{\text{opposite side}}{\text{hypothenuse}} \qquad cosa = \frac{\text{adjacent side}}{\text{hypothenuse}}$$

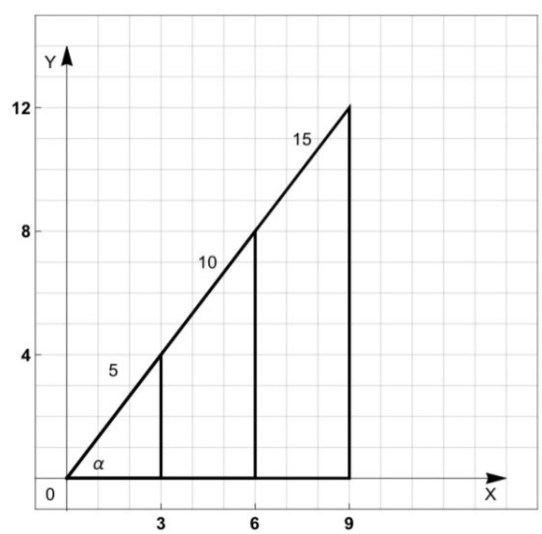
This is because the definitions on ***sin*** and ***cos*** do not really depend on size of the triangle, but only the angle itself. Since any two right triangles with the same angles are similar, it shows that if we have a right triangle with angle **α** and hypotenuse **c**, then the sides will be $c \sin \alpha$ and $c \cos \alpha$:



$$sina = \frac{\text{opposite side}}{\text{hypothenuse}} = \frac{c\ sina}{c}$$

$$cosa = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{c\ cosa}{c}$$

Example: Consider the angle a in the following triangles:



$$sina = \frac{\text{opposite side}}{\text{hypothenuse}} = \frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$

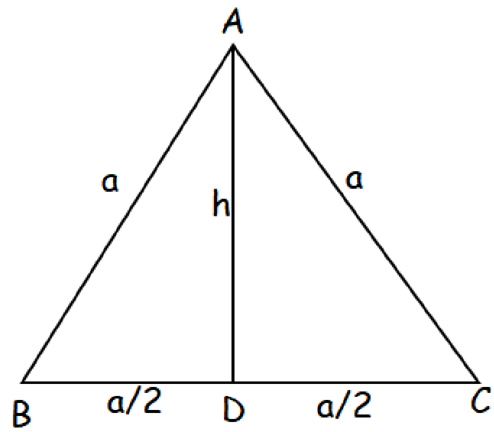
$$cosa = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{3}{5} = \frac{6}{10} = \frac{9}{15}$$

2. Table with values for trigonometric functions

Function	Notation	Definition	0^0	30^0	45^0	60^0	90^0
sine	sin(a)	$\frac{\text{opposite side}}{\text{hypothenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	cos(a)	$\frac{\text{adjacent side}}{\text{hypothenuse}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Homework problems

1. As we discussed in class, please find:



$sin(\angle B)$,

$cos\ (\angle B)$,

$sin(\angle BAD)$,

$cos(\angle BAD)$

2. Which one is greater?
   a. 0 or sin 0^0
   b. 1 or sin 30^0
   c. sin 45^0 or cos 45^0
   d. cos 60^0 or sin 30^0

3. Plot these functions, clearly define asymptotes:
   a. $y = \frac{1}{x+3} - 3$
   b. $y = \frac{1}{3-x} - 3$
   c. $y = x - \frac{1}{x}$